Shortest Paths

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Foundations of Data Science
BDA2121

Shortest Path Tree

Definition

Given an edge-weighted digraph and a designated vertex s, a shortest-paths tree (SPT) is a subgraph containing s and all the vertices reachable from s that forms a directed tree rooted at s such that every tree path is a shortest path in the digraph.

Dijkstra's Algorithm

The graph

- vertices
- weighted edges that connect the nodes: (u, v) denotes its weight.

Initialize

- *dist*, an array of distances from the sources node s to each node in the graph with $dist(s) = 0$; and all other nodes v $dist(v) = \infty$
- Q, a que of all nodes in the graph.
- S, an empty set.

Dijkstra's algorithm

Contd.

Algorithm

1. While Q is not empty, add node v, that is not already in S, from Q with the smallest $dist(v)$.
2. Add v to S.
3. Update $dist$ value of adjacent nodes of the current node v as follows:
   - 3.1 If $dist(v) + dist(u, v) < dist(u)$, update $dist(u)$ to the new minimal distance value.
   - 3.2 Otherwise, no updates.