Justify all your answers. Keep your answers to the point. Submit your work in groups of three.

1. How would you show that not every number of the form $N = (p_1.p_2.p_3 \dots p_n)+1$, where $p_1, p_2, p_3, \dots p_n, \dots$ is the list of all prime numbers?

2. The following notice was posted on the wall of a hospital emergency room:
NO HEAD INJURY IS TOO TRIVIAL TO IGNORE.
Reformulate to avoid the unintended second reading. (The context for this sentence is so strong that many people have difficulty seeing there is an alternative meaning.)

3. Official documents often contain one or more pages that are empty apart from one sentence at the bottom:
This page intentionally left blank.
Does the sentence make a true statement? What is the purpose of making such a statement? What reformulation of the sentence would avoid any logical problems about truth?

4. What strategy would you adopt to show that the conjunction $\phi_1 \wedge \phi_2 \wedge \dots \phi_n$ is true?

5. What strategy would you adopt to show that the conjunction $\phi_1 \wedge \phi_2 \wedge \dots \phi_n$ is false?

6. In the following table, T denotes 'true' and F denotes 'false'. The first two columns list all the possible combinations of values of T and F that the two statements ϕ and ρ can have. The third column should give the truth value (T or F) $\phi \wedge \rho$ achieves according to each assignment of T or F to ϕ and ρ.

ϕ	ρ	$\phi \wedge \rho$
T	T	?
T	F	?
F	T	?
F	F	?

7. Is it possible for one of $(\phi \wedge \rho) \vee \theta$ or $\phi \vee (\rho \vee \theta)$ to be true and the other false, or does the associative property hold for disjunction? Prove your answer.

8. What is the negation of *All foreign cars badly made.*

9. Let D be the statement "The dollar is strong", Y the statement "The Yuan is strong," and T the statement "New US–China trade agreement signed". Express the main content of each of the following (fictitious) newspaper headlines in logical notation. (Note that logical notation captures truth, but not the many nuances and inferences of natural language.) Be prepared to justify and defend your answers.
(a) Dollar and Yuan both strong
(b) Trade agreement fails on news of weak Dollar
(c) Dollar weak but Yuan strong, following new trade agreement
(d) Strong Dollar means a weak Yuan
(e) Yuan weak despite new trade agreement, but Dollar remains strong
(f) Dollar and Yuan can't both be strong at same time.
(g) If new trade agreement is signed, Dollar and Yuan can't both remain strong
(h) New trade agreement does not prevent fall in Dollar and Yuan
(i) US–China trade agreement fails but both currencies remain strong
(j) New trade agreement will be good for one side, but no one knows which.

10. Fill the following truth table. Provide a justification for your entries.

ϕ	ρ	$\phi \implies \rho$
T	T	?
T	F	?
F	T	?
F	F	?

11. Complete the following truth table. Justify your response.

ϕ	$\neg\phi$	ρ	$\phi \implies \rho$	$\neg\phi \vee \rho$
T		T		
T		F		
F		T		
F		F		

12. Verify the equivalences of the following two statements by means of a logical argument:
    (a) $\phi \implies (\rho \vee \theta) \equiv (\phi \implies \rho) \wedge (\phi \implies \theta)$
    (b) $(\phi \vee \rho) \implies \theta \equiv (\rho \implies \theta) \wedge (\rho \implies \theta)$

13. Wite the contrapositive and converse of the following statements:
    (a) If two rectangles are congruents, they have the same area.
    (b) If a triangle with sides a, b, c (c largest) is right angles, then $a^2 + b^2 = c^2$
    (c) If $2n - 1$ is prime, then n is prime.
    (d) If the Yuan rises, the Dollar will fall.

14. Express the following in symbolic form, using quantifiers for people:
    (a) Everybody loves somebody.
    (b) Everyone is tall or short.
    (c) Everyone is tall or everyone is short.
    (d) Nobody is at home.
    (e) If John comes, all the women will leave.
    (f) If a man comes, all the women will leave.

15. Show that $\neg[\exists x A(x)] \equiv \forall x[\neg A(x)]$

16. Prove that *There is an even prime bigger than 2* is false.

17. Prove that $\sqrt{2}$ is irrational. (Hint: contradiction).