MCMC and MRFs

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ML 2 BDA3321

Markov Random Field

An undirected graphical model is also known as Markov Randon Fields.
These are very useful in image and spatial analysis.

Examples of MRFs

1. **Ising Model**
   This is a model that arose from physics, used to model the behaviour of magnets.
2. **Hopfield Networks**
   This is an extension of the Ising model. The main application of Hopfield Network is in pattern completeion. This can also be interpreted as RNN.
3. **Potts Model**
   This model is commonly used in image segmentation.

Monte Carlo Principle

The Monte Carlo Principle states that if you take iid samples $x^{(i)}$ from an unknown high dimensional distribution $p(x)$, then as the number of sample gets larger the sample distribution will converge to the true distribution.

The Metropolis-Hastings Algorithm

- Given an initial value x_0
- Repeat:
  - Sample x^* from $q(x_i|x_{i-1})$
  - Sample u from the uniform distribution.
  - If $u < min(1, \frac{\tilde{p}(x^*)q(x^{(i)}|x^*)}{\tilde{p}(x^i)q(x^*)|x^i)})$
    set $x[i+1] = x^*$
  - Otherwise
    set $x[i+1] = x[i]$
- Until you have enough samples.

Gibbs Sampling

- For each variable x_j:
  - Initialize $x_j^{(0)}$
- Repeat:
  - For each variable x_j:
    - sample $x_1^{(i+1)}$ from $p(x_1|x_2^{(i)}, \ldots, x_n^{(i)})$
    - sample $x_2^{(i+1)}$ from $p(x_1 2|x_1^{(i)}, x_3^{(i)} \ldots, x_n^{(i)})$
    - $\ldots$
    - sample $x_n^{(i+1)}$ from $p(x_n|x_1^{(i)}, \ldots, x_{n-1}^{(i)})$
- Until you have enough samples.

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