

Going Backwards

Back-Propagation of Error

Error Function

Sum-of-squares errors is used.

$$E(\mathbf{t}, \mathbf{y}) = \frac{1}{2} \sum_{k=1}^N (y_k - t_k)^2$$

Activation Function

As an example; consider the sigmoid function:

$$a = g(h) = \frac{1}{1 + \exp(-\beta h)}$$

DIFFERENTIABILITY is key!

Back Propagation

The problems

1. For the **output** neurons, we don't know the inputs.
2. For the **hidden** neurons, we don't know the targets.
3. For **extra** hidden layers, we don't know the inputs or targets.

The Multilayer Perceptron Algorithm

Initialization

- ▶ Initialize all weights to small random values.

The Multi-layer Perceptron Algorithm

Training: Forwards phase

▶ repeat

For each input vector:

Forwards phase:

- ▶ Compute the activation of each neuron j in the hidden layer.
- ▶ Work through the network until you get to the output layer neuron.

The Multi-layer Perceptron Algorithm

Training: Backwards phase

Backwards phase:

- ▶ Compute the error at the output using:

$$\delta_o(\kappa) = (y_\kappa - t_\kappa)y_\kappa(1 - y_\kappa)$$

- ▶ Compute the error in the hidden layer(s) using:

$$\delta_h(\zeta) = a_\zeta(1 - a_\zeta) \sum_{\kappa=1}^N w_{\zeta\kappa} \delta_o(\kappa)$$

- ▶ Update the output layer weights using:

$$w_{\zeta\kappa} \leftarrow w_{\zeta\kappa} - \eta \delta_o(\kappa) a_\zeta^{\text{hidden}}$$

- ▶ Update the hidden layer weights using:

$$v_l \leftarrow v_l - \eta \delta_h(\kappa) x_l$$

MLP: Example

CNNs

References I

- [Mar14] Stephen Marsland. *Machine Learning, An Algorithmic Perspective*. CRC Press, 2014.
- [Ian17] Aaron Courville Ian Goodfellow Yoshua Bengio. *Deep Learning*. MIT Press, 2017.