

Advanced Statistical Methods
BCADA4321
Assingment 2

Date: 25.04.2022

Due on: 05.05.2022

Solve the following problems. Show your work. Submit in group of 3.

1. Let S be the set of integers. If $a, b \in S$, define aRb if $a + b$ is even. Prove that R is an equivalence relation. Determine the equivalence classes of S
(aRb means a and b are related under the relation R .
The set $[a] = \{x \in S | a \sim b\}$ is the equivalence class of a) (1)

2. For the following functions, determine if they are one-one or onto. Are any of them bijective?

- $f : \mathbb{Z} \rightarrow \mathbb{Z}$ such that $x \mapsto x^3$
- $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $x \mapsto x^3$
- $f : \mathbb{Z} \rightarrow \mathbb{N}$ such that $x \mapsto |x|$
- $f : \mathbb{Z} \rightarrow \mathbb{Z}$ such that $x \mapsto x^2$

(1)

3. Write the binary operation table for the set \mathbb{Z}_5 for the operation addition modulo 5.
($\mathbb{Z}_n = \{1, 2, \dots, n\}$) (1)

4. Determine which of the following form a group. Justify your answer.

- Rational numbers under ordinary addition.
- Real numbers under ordinary multiplication.
- Set of 3×3 real matrices under matrix multiplication.
- \mathbb{Z}_5 under addition modulo 5.

(1)

5. **Bonus Question** How can you; by either adding or deleting element from \mathbb{R} ; make \mathbb{R}^* a group under ordinary multiplication? (\mathbb{R}^* is the set obtained after you are done adding or deleting elements to \mathbb{R} , \mathbb{R} is the set of all real numbers.)