Advanced Statistical Methods
BCADA2221
Assingment 3

Date: 10.05.2022 **Due on: 17.05.2022**

Solve the following problems. Show your work. Submit your assingments in groups of three.

1. How many 3-digit numbers can be formed from the digits $1, 2, 3, 4, 5$ assuming that
   (a) repition of digits is allowed?
   (b) repition of digits is not allowed?

2. How many 3 digit even numbers can be formed from the digits $1, 2, 3, 4, 5, 6$ if the digits can be repeated?

3. A coin is tossed 3 times, and outcomes recorded. How many possible outcomes are there?

4. In how many ways can the letters of the word PERMUTATION be arranged if the
   (a) words start with P and end with S
   (b) vowels are all together
   (c) there are always 4 letters between P and S

5. A group consists of of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has
   (a) no girl?
   (b) atleast one boy and one girl?
   (c) at least 3 girls?

6. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted both NCC and NSS. If one of the students is selected at random, find the probability that
   (a) The student opted for NCC or NSS
   (b) The student has opted neither NCC nor NSS
   (c) The student has opted NSS but not NCC

7. Prove $E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i)$. Assume X_is are an iid sample.

8. Let X be a binomial random variable; with pmf given by

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots n$$

   where n is a positive integer, $0 \leq p \leq 1$. Find $E(X)$ and $V(X)$ (Use the definition from the last question).

9. Let X_1, X_2 be two independent random variables variabel and let a, b, c be constants. then prove that
   (a)

$$E(aX_1 + bX_2 + c) = aE(X_1) + bE(X_2) + c$$

   .

   (b) if $x \geq 0 \quad \forall x$ then, $E(X) \geq 0$

10. Let variance of a random varible be defined as

$$Var(X) = E(X - E(X))^2$$

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Let X be a random poisson variable with paramater λ. Assume $E(X) = \lambda$. Find $V(X)$. (Hint: $\sum_{i>0} \frac{1}{i!}\lambda^i = e^\lambda$)