Endomorphism Algebras of Sources of Simple Modules

Jayati Kaushik
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Supervised by Jun.-Prof. Dr. Caroline Lassueur
Department of Mathematics
Technische Üniversität Kaiserslautern

Declaration

I hereby declare that this thesis is my own work and that I have acknowledged the sources where relevant.

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Kaiserslautern
Jayati Kaushik

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Chapter 1

Introduction

If G is a finite p-group, K a field of characteristic p. Then, a KG-module M is an endo-permutation module if $\mathrm{End}_K(M) \cong M \otimes_K M^*$ has a basis X invariant under the action of G. In 1978, E. Dade introduced the concept of endo-permutation modules in the field of modular representation theory (see [Dad78b] and [Dad78a]). The study of these modules is important in the field of representation theory as they appear very often. They appear as sources of irreducible simple modules of groups. In particular, they appear as sources of simple modules for p-solvable groups (See [Pui88] and [Thé95]). More precisely L. Puig proved that this source must be isomorphic to the cap of an endo-permutation module of the form

$$\bigotimes_{Q/R \in S} Ten_Q^P Inf_{Q/R}^Q (M_{Q/R}),$$

where $M_{Q/R}$ is an simple torsion endo-trivial module with vertex Q/R, and S is a set of cyclic, quaternion and semi-dihedral sections of the vertex of the simple KG-module([Maz03]). In the paper [Maz03], a method to explicitly realize the cap of any such simple module as the source of a simple module for a finite p-nilpotent group is given.

In contrast, here we focus on the sources of simple modules of finite simple groups. In particular, we want to see when these sources are endo-permutation. This thesis is a computational work where we look at sources of simple modules of simple groups over fields of different charactersitics. Furthermore, we calculate the respective sources and check if they are endo-permutation. We use both theoretical arguments and computer programs to achieve this.

The thesis is structured as follows. In Chapter 2, we introduce all the theoretical background required for our work. We start from the notion of relative projectivity, then define the vertices and sources of simple modules and their properties, then define endo-permutation modules and show the algorithmic ideas involved, and finally describe block theory and it's implications on our aim. In Chapter 3, we look at the sources of simple modules of sporadic groups $M_{11}, M_{12}, M_{22}, M_{23}, J_1$ and J_2. In Chapters 4, Chapter 5 and Chapter 6, we study some more simple groups. Finally, in Chapter 7 we summarize our observations and make conjectures based on the data collected.

Chapter 2

Preliminaries

In this chapter, we want to introduce all the theoretical background required for this thesis. We start by talking about modules, then move to relatively projective modules and then discuss vertices and sources of modules. After this, we discuss endo-permutation modules. Finally, we discuss block thoery.

2.1 Notation

Unless otherwise mentioned, G is a finite group, p is a prime number, K is a field of characteristic p, M is a KG-module, H is a p-subgroup of G, P denotes a finite p-group, V a vertex and S denotes a source of KG-module M

2.2 Relative Projectivity

For this chapter, we refer to [Web16] Chapter 11 and Chapter 12 and [Alp86] Chapter 2 and Chapter 4. In the context of modular representation theory of groups, the relationship between the representations of groups and its subgroups is seen in the notion of relative projectivity.

Definition 2.2.1. Let H be a subgroup of G and K a field. A KG-module is said to be **H-free** if it has the form $N \uparrow_H^G$ for some KH-module N. It is **H-projective** or **projective relative to** H, if it is a direct summand of a module of the form $N \uparrow_H^G$ for some KH-module N.

Theorem 2.2.1. *Let G be a finite group with subgroup H. Then the following are equivalent for a KG-module U*

1. *The KG-module U is H-projective.*

2. *Whenever $\phi : V \to U$ is a homomorphism of RG-modules such that $\phi\downarrow_H^G$ commutative ring with 1 : $V\downarrow_H^G \to U\downarrow_H^G$ is a split epimorphism of KH-modules, then ϕ is a split epimorphism of KG-modules.*

3. *The KG-module U is a direct summand of $U\downarrow_H^G\uparrow_H^G$.*

Theorem 2.2.2. *Suppose that H is a subgroup of G for which $|G\colon H|$ is invertible in the field K, and let U be a KG-module. Then U is projective as a KG-module if and only if $U\downarrow_H^G$ is projective as a KG-module.*

Proof. We know that if U is projective then $U\downarrow_H^G$ is projective, no matter what the subgroup H is.
Conversely, if $U\downarrow_H^G$ is projective it is a direct summand of a free module KH^n for some $n \in \mathbb{N}$. Since U is H-projective it is a direct summand of $U\downarrow_H^G\uparrow_H^G$, which is a direct summand of $KH^n\uparrow_H^G \cong KG^n$. Therefore, U is projective. □

2.3 Vertices and Sources

In what follows, we explore some techniques that are useful in understanding simple modules better. In particular, we discuss in some detail the vertices and sources of modules, and their properties.

Theorem 2.3.1. *Let K be a field, and let U be an simple KG-module.*

1. *There is a unique conjugacy class of subgroups Q of G that are minimal subject to the property that U is Q-projective.*

2. *Let Q be a subgroup of G, that is minimal such that U is Q-projective. There is an indecomposable KQ-module S that is unique up to conjugacy by elements of $N_G(Q)$ such that U is a direct summand of $S\uparrow_Q^G$. Such a module S is necessarily a direct summand of $U\downarrow_Q^G$.*

Proof. 1. Assume that U is both H-projective and J-projective where H and J are subgroups of G. Then, U is a direct summand of $U\downarrow_H^G\uparrow_H^G$ and also of $U\downarrow_J^G\uparrow_J^G$, so it is also a direct summand of

$$U\downarrow_H^G\uparrow_H^G\downarrow_J^G\uparrow_J^G = \bigoplus_{g\in[J\backslash G/H]} ({}^g((U\downarrow_H^G)\downarrow_{J^g\cap H}^H)\uparrow_{J\cap {}^gH}^J)\uparrow_J^G$$

$$= \bigoplus_{g\in[J\backslash G/H]} ({}^g(U\downarrow_{J^g\cap H}^G))\uparrow_{K\cap {}^g|H}^G$$

using transitivity of restriction and induction. Hence U must be a direct summand of some module induced from one of the groups $J \cap^g H$. If both H and J happen to be minimal subject to the condition that U is projective relative to these groups, we deduce that $J \cap^g H = J$, so $J \subseteq {}^gH$. Similarly $H \subseteq {}^{g'}J$ for some g' and so J and H are conjugate.

2. Let Q be a minimal subgroup reletive to which U is projective. We know that U is a direct summand of $U\downarrow_Q^G\uparrow_Q^G$ and hence it is a direct summand of $T\uparrow_Q^G$ for some indecomposable summand T of $U\downarrow_Q^G$. Suppose that T' is another indecomposable module for which U is a direct summand of $T'\uparrow_Q^G$. Now T is a direct summand of

$$T'\uparrow_Q^G\downarrow_Q^G = \bigoplus_{g\in[Q\{\}G/Q]} ({}^g(T'\downarrow_{Q^g\cap Q}))\uparrow_{Q\,\cap\,{}^gQ}^Q$$

and hence a direct summand of some $({}^g(T'\downarrow_{Q^g\cap Q}))\uparrow_{Q\,\cap\,{}^gQ}^Q$. For this element g we deduce that U is $Q\cap^g Q$-projective and by minimality of Q we have $Q = Q\cap^g Q$ and $g \in N_G(Q)$. Now T is a direct summand of ${}^gT'$, and, since both modules are indecomposable, we have $T =^g T'$. We deduce from the fact that T is a direct summand of $U\downarrow_Q^G$ that $T' = {}^{g^{-1}}T$ must be a direct summand of $({}^{g^{-1}}U)\downarrow_Q^G$ and hence of $U\downarrow_Q^G$, since ${}^{g^{-1}}U \cong U$ as KG-modules.

□

Definition 2.3.1. A subgroup Q of G, minimal, relative to which the indecomposable module U is projective is called a **vertex** of U, and it is defined up to conjugacy in G. A KQ-module S for which U is a direct summand of $S\uparrow_Q^G$ is called a **source** of U and, given the vertex Q, it is defined up conjugacy by elements of $N_G(Q)$.

We record some properties of vertex.

Theorem 2.3.2. *Let K be a field of characteistic p.*

1. *The Vertices of every indecomposable KG-module are p-subgroups of G.*

2. *An indecomposable KG-module is projective if and only if it's vertex is ¡1¿.*

3. *The vertices of the trivial KG-module K are Sylow p-subgroups of G.*

Proof. 1. We know from Theorem 2.3.1 that every module is projective relative to a Sylow p-subgroup, and so vertices must be p-subgroups.

2. If an indecomposable module is projective, it is a direct summand of KG, which is induced from 1, so it must be free as a K-module and we have vertex $< 1 >$.
   Conversely, if U has vertex $< 1 >$ it is a direct summand of $U\downarrow^G_{<1>}\uparrow^G_{<1>}$, so if U is free as a K-module it is a direct summand of $K\uparrow^G_{<1>}= KG$ and hence is projective.

3. Let Q be a vertex of K and P a Sylow p-subgroup of G containing Q. Then K is a direct summand of $K\uparrow^P_{P\cap^g Q}$ for some $g \in G$. We claim that for every subgroup $H \leq P$, $K\uparrow^P_H$ is an indecomposable KP-module. From this it follows that $K = K\uparrow^P_{P\cap^g Q}$ and that $Q = P$. The only simple KP-module is K, and

$$Hom_{KP}(K\uparrow^P_H, K) = Hom_{RH}(K, K) \cong K$$

   is a space of dimension 1. This means that $K\uparrow^P_H$ has a unique simple quotient, and hence is indecomposable.

□

The following is an interesting property of vertices that will be useful later.

Theorem 2.3.3. *Let K be a field of characteristic p and let P be a Sylow p-subgroup of the group G. Let V be an indecomposable KG-module with vertex $Q \subseteq P$. Then $|P : Q|$ divides $dim_K V$.*
As a special case, it follows that if p does not divide $dim_K V$, then P itself is a vertex of V.

Theorem 2.3.4. *(Green Correspondence) Let K be a field of characteristic p. Let Q be a p-subgroup of G and L a subgroup of G that contains the normalizer $N_G(Q)$.*

1. *Let U be an indecomposable RG-module with vertex Q. Then in any decomposition of $U\downarrow^G_L$ as a direct sum of indecomposable modules there is a unique indecompoasable direct summand $f(U)$ with vertex Q.*
   Writing $U\downarrow^G_L= f(U) \oplus X$, each direct summand of X is projective relative to a subgroup of the form $L \cap^x Q$ where $x \in G - L$.

2. *Let V be an indecomposable KL-module with vertex Q. Then any decomposition of $V\uparrow^G_L$ as a direct sum of indecomposable modules there*

is a unique indecomposbale summand $g(V)$ with vertex Q. Writing $V\uparrow_L^G = g(V) \oplus Y$, each direct summand of Y is projective relative to a subgroup of the form $Q \cap {}^xQ$ where $x \in G - L$.

3. *In the notation of parts (1) and (2) we have*

$$gf(U) \cong U$$

and

$$fg(V) \cong V.$$

2.4 Endo-permutation Modules

In this section we refer to [Thé07] section 2 for our theory. We are concerned with modules over the group algebra KP. Group algebra KP is finitely generated since $|P| \leq \infty$. It was shown that the family of endo-permutation modules is small enough to be classified and large enough to be useful. Studying these is particularly useful in our scenario because they appear as sources of simple modules for p-solvable groups.

Definition 2.4.1. A KP-module M is called a **permutation module** if it has a basis X which is invariant under the action of P.

We write $M = KX$. The finite P-set X decomposes as a disjoint union of orbits and each orbit is isomorphic to a set of cosets P/Q for some subgroup Q of P. Thus, KX decomposes accordingly as a direct sum of submodules of the form $K[P/Q]$. Every such module $K[P/Q]$ is indecomposable. Therefore, the indecomposable permutation KP-modules are parametrized by the subgroups of P up to conjugation. In particular, there are finitely many of them. Since $K[P/Q] \cong Ind_Q^P(K)$, its vertex is Q, and therefore the only decomposable permutation module with vertex P is the trivial module K. We note that any direct summand of a permutation KP-module is again a permutation module.

Definition 2.4.2. A KP-module M is called an **endo-permutation module** if $\mathrm{End}_K(M)$ is a permutation module.

In the defition above, $\mathrm{End}_K(M) \cong M \otimes_K M^*$ as a KP-module, where $M^* = Hom_K(M, K)$ is the dual module and the tensor product is over K. $\mathrm{End}_K(M)$ is endowed with its natural KP-module structure coming from the action of P by conjugation: if $g \in P$ and $\phi \in \mathrm{End}_K(M)$, then ${}^g\phi(m) = g \cdot \phi(g^1 \cdot m)\ \forall\, m \in$

M.
It follows that M is an endo-permutation module if and only if $M \otimes_K M^*$ has a P-invariant basis.

Definition 2.4.3. A KP-module M is called **endo-trivial** if there exists a projective KP-module F such that $\mathrm{End}_K(M) \cong K \oplus F$ as a KP-module.

It is clear that any endo-trivial module is an endo-permutation module since every projective KP-module is free and since a free KP-module is a direct sum of copies of KP, which has an obvious P-invariant basis. This can be stated in the form of a theorem as follows:

Theorem 2.4.1. *Let M be a KP-module such that it is an endo-trivial module. Then, M is an endo-permutation KP-module.*

The following theorem lists some properties of endo-permutation modules.

Theorem 2.4.2. *The class of endo-permutation modules contains the permutation modules. Moerover, it is closed under taking direct summands, duals, tensor products and restrictions to a subgroup.*

Proof. Let M, M_1, M_2 be endo-permutation KP-modules. We have that

$$End_K(M) \cong M \otimes_K M^* \cong M^{**} \otimes_K M^* \cong End_K(M^*)$$

Hence endo-permutation modules are closed under taking duals.
We have the isomorphism

$$End_K(M_1 \otimes_K M_2) \cong End_K(M_1) \otimes_K End_K(M_2)$$

This combined with the fact that permutation modules are closed under tensor products gives us that endopermutation modules are closed under taking tensor products.
Let W be a KP-module such that it is a direct summand of M. Then, $\mathrm{End}_K(W)$ is a direct summand of $\mathrm{End}_K(M)$. Since permutation modules are closed under direct summands, it follows that endo-permutation modules are closed under taking direct summands.
Let Q be a subgroup of P. Consider $M \downarrow_Q^P$. It is clear that a P -invariant basis of $\mathrm{End}_K(M)$ is also Q-invariant. Hence endo-permutation modules are closed under restriction to a subgroup. □

The class of endo-permutation modules is not closed under direct sums (i.e. If M_1 and M_2 are endo-permutation modules, then $M_1 \oplus M_2 \ncong X$ where X is an endo-permutation module), and induction to overgroups. The natural operation to be used is hence tensor product instead of direct sums.

2.5 Sources as Endo-permutation Modules

In this thesis we want to investigate the sources of simple modules. In particular, we want to know if and when these sources are endo-permutation modules. To accomplish this, we start by stating the following theorem:

Theorem 2.5.1. *Let G be a finite group, K a filed of characteristic p and assume that $p \mid |G|$. Let P be a p-subgroup of G such that P is the vertex of a simple KG-module S. Let V be a KP-module that is a source of S. Then we have the following:*
If $End_K(S\downarrow_P^G) \cong (S\downarrow_P^G)^* \otimes_K (S\downarrow_P^G)$ is a permutation KP-module, then so is $End_K(V)$.

Proof. We have from Theorem 2.3.1 $S\downarrow_P^G \cong V \oplus X$ for some KP-module X. We know that $(S\downarrow_P^G)^* \otimes_K (S\downarrow_P^G)$ is an endo-permutation module.

$$(S\downarrow_P^G)^* \otimes_K (S\downarrow_P^G) \cong (V \oplus X)^* \otimes_K (V \oplus X)$$
$$\cong V^* \otimes_K V \oplus V^* \otimes_K X \oplus X^* \otimes_K V \oplus X^* \otimes_K X$$

We see that $V^* \otimes_K V$ is a direct summand of $(S\downarrow_P^G)^* \otimes_K (S\downarrow_P^G)$. Then, from the Theorem 2.4.2 it must be that $V^* \otimes_K V$ is a permutaion KP-module. □

In view of the Theorem 2.5.1, we are now able to do some calculations as follows:

Let G be a finite group, K an algebraically closed field with characteristic p and P a Sylow p-subgroup of G. Let S be a simple KG-module with vertex P (For example, see Theorem 2.3.3). Then we test whether

$$(S\downarrow_P^G)^* \otimes_K (S\downarrow_P^G) \cong X$$

where X is a KP-permutation module.

This can be achieved using the computer algebra system MAGMA, and we show our code below:

```
G:= Group; /* loading required group */
S:=simpleModules(G,GF(l)); */l is the characteristic of the field.
S;/* we calculate and display the simple KG-modules*/
verts:= [Vertex(s): s in S];
[#v : v in verts];
```

```
verts;/* Vertex calculates the Vertices of the simple modules*/
L1:=Restriction(S[i],verts[i]);
L1;/*L1 is the required restriction of the modules.
We use the command Restriction()*/
D1:=Dual(L1);
D1;
T1:=TensorProduct(D1,L1); T1;
/* The command Dual() calculates the dual of the module,
command TensorProduct() to calculate the tesor product*/
IsPermutationModule(T1);
```

In the code above, we first calculate $(S \downarrow_P^G)^* \otimes_K (S \downarrow_P^G)(\cong End_K(S \downarrow_P^G))$ and then use IsPermutationModule() to check if $(S \downarrow_P^G)$ is an endo-permutatioin module. In case when IsPermutaitonModule() returns a false value, we calculate the source of the simple module under investigation, and check to see if the source is an endo-permutation module. It is important to note here that while calculating the sources, we must take a field large enough for accurate computation of the field. This is taken care of in all cases when calculating the source explicitly was required.

Converse if the Theorem 2.5.1 can be stated as follows:

Theorem 2.5.2. *Let G be a finite group, K a filed of characteristic p and assume that $p \mid |G|$. Let P be a p-subgroup of G such that P is the vertex of a simple KG-module S. Let V be a KP-module that is a source of S. Then If $End_K(S \downarrow_P^G) \cong (S \downarrow_P^G)^* \otimes_K (S \downarrow_P^G)$ is a not a permutation KP-module, then neither is $End_K(V)$.*

We note here that the converse of Theorem 2.5.1 is not true; as will be seen in the later chapters. Where it is not true is shown in tables in the following chapters by writing yes* in the tables.

2.6 Block Theory

We describe very briefly block theory as it pertains to our needs in this thesis. Let K be a field and A be a finite dimensional K-algebra.

2.6.1 Blocks

Theorem 2.6.1. *The algebra A has a unique decomposition*

$$A = A_1 + \cdots + A_r$$

into a direct sum of ideals, each of which is indecomposable as an algebra.

Proof. Express $A = A_1 + \dots A_r$ as a direct sum of ideals, each of which is inddecomposable as an algebra. Let B be such a summand in another such decomposition. It suffices to prove that $B = A_i$, for some i, $1 \leq i \leq r$. Let $b \in B$ and express $b = a_1 + \cdots + a_r$, each $a_i \in A_i$. Since A_i has a unit element, the component of the unit element 1 of A in A_i, and B is an ideal, it follows that $a_i \in B$ for each i. Hence,

$$B = (B \cap A_1) + \cdots + (B \cap A_r)$$

and this is a decomposition of B, inasmuch as each $B \cap A_i$ is an ideal of B. By the assumption on B we must have that all the summands; except one; in the expression of B are zero. Hence, $B \subseteq A_i$ for some i. However, by the same argument, applied to A_i in comparison with the decomposition of A involving B, we must have $A_i \subseteq B$ and hence the theorem. □

The above theorem motivates the following definition.

Definition 2.6.1. Let A be a K-algebra such that it's unique decomposition into a direct sum of two sided ideals is given by

$$A = A_1 \oplus \cdots \oplus A_r.$$

Then the subalgebras $A_1, \dots, A_r$ are called the **blocks** of A.

With the notation $A = A_1 + \dots A_r$ for the decomposition of A into blocks. If M is an A-module, $A_i M = M$ and $A_j M = 0$ for all $i \neq j$, then we say that M **lies in the block** A_i. Conversely, suppose that N is an A_i-module; we can make N into an A-module by simply demanding that each $A_j, j \neq i$, annihilates N, and it lies in A_i. Each A-module lying in A_i clearly arises in this way.
There is another way to express this. Let $1 = e_1 + \cdots + e_r$ be the components of the unit element 1 so that e_j is an element of A_j. In particular, since 1 induces the identity on each A-module, if M lies in A_i, then e_i is the identity on M and each $e_j, j \neq i$, annihilates M. On the other hand, if this occurs then M is in A_i, as $AM = Ae_i M = A_i M$ and $A_j M = A_j e_j M$. This criterion for lying in a block immediately implies that submodules, quotient modules and direct sums of modules lying in a block A_i also lie in A_i. Moreover, if M_i and M_j lie in blocks A_i and A_j respectively, and $i \neq j$ then $Hom_A(M_i, M_j) = 0$. Indeed, if $\phi \in Hom_A(M_i, M_j)$ then e_i is the identity on $\phi(M_i)$ but $e_i M_j = 0$.

Theorem 2.6.2. *If M is am A-module then M has a unique direct sum decomposition*

$$M = M_1 \oplus \cdots \oplus M_r$$

where M_i lies in the block A_i.

Proof. We set $M_i = A_i M$ so certainly M_i is an A-module and lies in the block A_i while M is the sum of $M_1, \dots, M_r$ since $1 \cdot M = M$. Moreover, the sum is direct. Indeed suppose that $m_1 + \cdots + m_r = 0$ where $m_i \in M_i$. Applying the unit element of A_j to this sum yields $m_j = 0$, for each $j, 1 \leq j \leq r$.
On the other hand, suppose $m = N_1 \oplus \cdots \oplus N_r$, is a direct sum where N_i lies in A_i. Thus, $N_i = A_i N_i \subseteq A_i M = M_i$ so $N_i \subseteq M_i$ and hence $N_i = M_i$, since M is the sum of $N_1, \dots, N_r$. □

2.6.2 Defect Groups

We shall now specialize to the case of group algebras. The key idea is to regard KG as a module for the group algebra $k[G \times G]$: if $a \in KG, g_1, g_2 \in G$ then set $(g_1, g_2)a = g_1 a g_2^{-1}$. Therefore, the submodules of the $K[G \times G]$-module KG are exactly the ideals of KG. In particular, KG has a unique decomposition into the direct sum of indecomposable $K[G \times G]$-modules; the direct summands are the blocks of KG. These blocks, as $K[G \times G]$-modules, are pairwise non-isomorphic: their annihilators in $K[G \times < 1 >] \subseteq K[G \times G]$ are different.

Theorem 2.6.3. *If B is a block of KG then B has a vertex, as a $K[G \times G]$-module, of the form δD, where D is a p-subgroup of G and*

$$\delta : G \to G \times G, g \mapsto (g, g).$$

Proof. It suffices to prove that the $K[G \times G]$-module B is relatively δG-projective since then δG contains a vertex of B. But B is a direct summand of the $K[G \times G]$-module KG, so we need only to demonstrate that KG is relatively δG- projective. But KG contains the subspace $K \cdot 1$, which is a $K[\delta G]$-module, in fact it is the trivial $K[\delta G]$- module. Moreover,

$$dim_K KG = |G| dim_K (K \cdot 1) = |G \times G \colon \delta(G)| dim_K (K \cdot 1)$$

and it is easy to see that $K \cdot 1$ generates the $K[G \times G]$-module KG. Hence, by our characterization of induced modules, $KG \cong (K \cdot 1)^{G \times G}$ so certainly KG is relatively $\delta(G)$-projective. □

Definition 2.6.2. Let B be a block of KG, $\delta : G \to G \times G, g \mapsto (g, g)$, and D be a p-subgroup of G, such that B has a vertex as a $K[G \times G]$-module of the form δG. Such subgroups D of G are called **defect groups** of B. If D has order p^d; where d is a positive integer; then B is said to be of **defect** d .

Theorem 2.6.4. *Let G be a finite group. K a field of characteristic p. Let M be an simple KG-module belonging to a block with defect 0. This implies that source of M is the trivial 1-dimensional module. This implies that source S of M is an endo-permutation module using Theorem2.4.1.*

Proof. We know that if the block has defect 0, then the module belonging to that block is projective. It implies the, that source is an endo-permutation module. □

Theorem 2.6.5. *Let M be an simple liftable KG-Module with a cyclic defect group with vertex V and KV-source S. Then S is an endo-permutaion module.*

Proof. [Lin96] Theorem 6.5. □

Chapter 3

Sporadic Groups

3.1 Notation

For all following chapters; unless otherwise mentioned, we refer to notations as listed below:

1. S_n is an simple KG-module of dimension n
2. S_n^* denotes the dual of S_n.
3. When we have to non-isomorphic non-dual modules of same dimension we distinguish them be adding index to their dimension, i.e, S_{n_1} and S_{n_2} are two non-isomorphic non-dual modules of dimension n.
4. V_m ia a vertex of S_n with order n. When nothing is mentioned, defect group is the vertex. Otherwise, the structure of vertices is displayed.
5. P_l is a source of S_n. It is a module of dimension l.
6. Yes* impies that these cases show that the converse of theorem 2.5.1 is not true.
7. $< 1 >$ denotes trivial group
8. C_n denotes cyclic group of order n
9. A_n denotes alternating group of degree n
10. $\mathfrak{S}_\mathfrak{n}$ denotes symmetric group of degree n
11. D_n denotes dihedral group of order n
12. Q_n denotes quaternion group of order n

13. QD_ndenotes quasidihedral group of order n
14. $A \times B$ denotes direct product of A and B.
15. $N\colon H$ denotes semidirect product
16. $\mathrm{Syl}_p(G)$ denotes set of sylow p-subgroups of group G.

Where nothing is mentioned, we used [Con+85] for information on the simple groups studied below. Since the source of S_1 is always trivial 1-dimensional module, which is endo-permutation, we sometimes omit it from our discussions, restricting it only to all simple modules except S_1. Unless otherwise mentioned, "all modules" is meant to be read as all the simple modules of the concerned group wherever we describe these modules.

3.2 Mathieu Groups

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The first five sporadic groups to be discovered were the five Mathieu groups M_{11}, M_{12}, M_{22}, M_{23} and M_{24}. They are named after the french mathematician Èmile Léonard Mathieu (1835-1890), who discovered these groups around 1860.

3.2.1 Mathieu Group M_{11}

The smallest of the five Mathieu groups is the group M_{11} with a cardinality of $|M_{11}| = 7920 = 2^4 \cdot 3^2 \cdot 5 \cdot 11$. It is generated by the two permutations $x = (1,4,3,8)(2,3,6,9)$ and $y = (2,10)(4,11)(5,7)(8,9)$.

Characteristic 2

Table 3.1: The Group M_{11} for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{16}	P_1	4	SD_{16}	Yes
S_{10}	Q_8	P_1	4	SD_{16}	Yes*
S_{16}	V_1	P_1	0	$< 1 >$	Yes
S_{16_1}	V_1	P_1	0	$< 1 >$	Yes
S_{44}	$C_2 \times C_2$	P_1	4	SD_{16}	Yes*

Here we have three cases: S_1, S_{16},S_{16_1} where we know without any calculations that source is endo-permutation. For S_1 this is the case since source of 1-dimensional module is trivial source, which is endo-permutation by Theorem 2.4.1. For S_{16_1} and S_{16_2} we know that source is endo-permutation because of Theorem 2.6.4. In all other cases, the defect group is the semi dihedral group of order 16, and vertices lie in $\mathrm{Syl}_2(M_{11})$; and source is the trivial source, and hence endo-permutation. That the trivial source is an endo-permutation module is made clear from the Theorem 2.4.1. Here, we also have instances to show that the converse of Theorem 2.5.1 is not true; indicated in the table by Yes*. In cases where our computation returned a false value for the command IsPermutationModule() in the code presented in the previous section, we calculated the source of such a module and checked to see if it is an endo-permutation module. Unless otherwise mentioned, these same arguments are used for all other groups studied in this thesis.

Characteristic 3

Table 3.2: The Group M_{11} for characteristic correct this 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_9	P_1	2	$C_3 \times C_3$	Yes
S_5	V_9	P_5	2	$C_3 \times C_3$	No
S_5^*	V_9	P_5	2	$C_3 \times C_3$	No
S_{10_1}	V_9	P_1	2	$C_3 \times C_3$	Yes*
S_{10_2}	V_9	P_{10}	2	$C_3 \times C_3$	No
$S_{10_2}^*$	V_9	P_{10}	2	$C_3 \times C_3$	No
S_{24}	V_9	P_3	2	$C_3 \times C_3$	No
S_{45}	V_1	P_1	0	$< 1 >$	Yes

Here, the defect group of all modules except S_{45} is the elementary-abelian group of order 9; in $\mathrm{Syl}_3(M_{11})$. In all these cases except S_{10_1} sources are not endo-permutation.

Characteristic 5

Table 3.3: The Group M_{11} for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{10_1}	V_1	P_1	0	$< 1 >$	Yes
S_{10_2}	V_1	P_1	0	$< 1 >$	Yes
$S^*_{10_2}$	V_1	S_1	0	$< 1 >$	Yes
S_{11}	V_5	P_1	1	C_5	Yes*
S_{16}	V_5	P_1	1	C_5	Yes*
S^*_{16}	V_5	P_1	1	C_5	Yes*
S_{45}	V_1	P_1	0	$< 1 >$	Yes
S_{55}	V_1	P_1	0	$< 1 >$	Yes

In this case, the defect group of S_1, S_{11},S_{16}, S^*_{16} is the cyclic group of order 5. All the vertices lie in $Syl_5(M_{11})$. sources in these case are endo-permutation, but appear as examples of the fact that the converse of 2.5.1 is not true. In all the other cases, since the defect group is the trivial group, we already know without any calculation that sources are endo-permutation.

Characteristic 11

In this case, we have only the trivial cases where sources are endo-permutation.

Table 3.4: The Group M_{11} for characteristic 11

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{11}	P_1	1	C_{11}	Yes
S_9	V_{11}	P_9	1	C_{11}	No
S_{10}	V_{11}	P_{10}	1	C_{11}	No
S^*_{10}	V_{11}	P_{10}	1	C_{11}	No
S_{10}	V_1	S_1	1	$< 1 >$	Yes
S_{16}	V_{11}	P_5	1	C_{11}	No
S_{44}	V_1	P_1	0	$< 1 >$	Yes
S_{55}	V_1	P_1	0	$< 1 >$	Yes

The non-trivial vertices lie in $\mathrm{Syl}_{11}(M_{11})$. The defect group; when not trivial; is the cyclic group of order 11. And in these cases, we see that sources are not endo-permutaiton.

3.2.2 Mathieu Group M_{12}

The next group we study is the Mathieu Group M_{12}. It is of order $|M_{12}| = 95040 = 2^6{\cdot}3^3{\cdot}5{\cdot}11$. It is generated by two permutation $x = (1,4)(3,10)(5,11)(6,12)$ and $y = (1,8,9)(2,3,4)(5,12,11)(6,10,7)$. It can also be generated by the permutation $a = (2,3)(5,6)(8,9)(11,12)$ and $b = (1,2,4)(3,5,7)(6,8,10)(9,11,12)$. This group forms the second smallest of the Mathieu Groups. We study the sources of all the irreducible Modules of this group for the characteristics $p \in \{2,3,5,11\}$, and present our results below.

Characteristic 2

Table 3.5: The Group M_{12} for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation
S_1	V_{64}	P_1	6	$(C2 \times C2 \times C2) : (C2 \times C2)) : C2$	Yes
S_{10}	V_{64}	P_{10}	6	$(C2 \times C2 \times C2) : (C2 \times C2)) : C2$	Yes
S_{16}	V_4	P_1	2	$Z/2Z \times Z/2Z$	Yes*
S_{16}^*	V_4	P_1	2	$Z/2Z \times Z/2Z$	Yes*
S_{44}	V_{64}	P_{44}	6	$(C2 \times C2 \times C2) : (C2 \times C2)) : C2$	No
S_{144}	V_4	P_1	2	$C_2 \times C_2$	Yes*

Here the vertices of S_1, S_{10}, S_{44} lie in $\mathrm{Syl}_2(M_{12})$. We see that only in the case of S_{44} sources is not endo-permutation. We also three examples; S_{16}, S_{16}^* and S_{144}; showing the converse of Theorem 2.5.1 is not true. Note that in these cases, source module is 1 dimensional.

Characteristic 3

Table 3.6: The Group M_{12} for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{27}	P_1	3	$(C3 \times C3) : C3$	Yes
S_{10_1}	V_{27}	P_{10_1}	3	$(C3 \times C3) : C3$	No
S_{10_2}	V_{27}	P_{10_2}	3	$(C3 \times C3) : C3$	No
S_{15_1}	V_{27}	P_{15_1}	3	$(C3 \times C3) : C3$	No
S_{15_2}	V_{27}	P_{15_2}	3	$(C3 \times C3) : C3$	No
S_{34}	V_{27}	P_7	3	$(C3 \times C3) : C3$	No
S_{45_1}	$C3 \times C3$	P_6	3	$(C3 \times C3) : C3$	No
S_{45_2}	$C3 \times C3$	P_6	3	$(C3 \times C3) : C3$	No
S_{45_3}	V_3	P_1	1	C_3	Yes*
S_{54}	V_1	P_1	0	$< 1 >$	Yes
S_{99}	V_3	$P1$	1	C_3	Yes*

Here vertices of S_1, S_{10_1},S_{10_2},S_{15_1},S_{15_2}, S_{32}, S_{45_1} and S_{45_2} lie in $\mathrm{Syl}_3(M_{12})$. These belong to the principal block with defect 3. In all these cases, sources are not endo-permutation. In all other cases, source are endo-permutation. S_{54} lies in a block with defect 0 and hence requires no computation. S_{45_3} and S_{99} illustrate the converse of theorem 2.5.1 is not true.

Characteristic 5

Table 3.7: The Group M_{12} for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{11_1}	V_5	P_1	1	C_5	Yes*
S_{11_2}	V_5	P_1	1	C_5	Yes*
S_{16_1}	V_5	P_1	1	C_5	Yes*
S_{16_2}	V_5	P_1	1	C_5	Yes*
S_{45}	V_1	P_1	0	$< 1 >$	Yes
S_{55_1}	V_1	P_1	0	$< 1 >$	Yes
S_{55_2}	V_1	P_1	0	$< 1 >$	Yes
S_{55_3}	V_1	P_1	0	$< 1 >$	Yes
S_{66}	V_5	P_1	1	C_5	Yes*
S_{78}	V_5	P_3	1	C_5	No
S_{98}	V_5	P_3	1	C_5	No
S_{120}	V_1	P_1	0	$< 1 >$	Yes

Here vertices of S_1,S_{11_1},S_{11_2},S_{16_1},S_{16_2},S_{66},S_{78} and S_{98} lie in $\mathrm{Syl}_5(M_{12})$. These lie in blocks with defect 3. Only for S_{78} and S_{98}, sources are not endo-permutation. In all other cases, source are 1 dimensional trivial modules and hence endo-permutation. Here again, we see in abundance illustration of the fact that the converse of theorem 2.5.1 is not true.
The modules S_{45}, S_{55_1}, S_{55_2}, S_{55_3} and S_{120} lie in blocks with defect 0 and hence require no computation.

Characteristic 11

Table 3.8: The Group M_{12} for characteristic 11

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{11}	P_1	1	C_{11}	Yes
S_{11_1}	V_1	P_1	0	$<1>$	Yes
S_{11_1}	V_1	P_1	0	$<1>$	Yes
S_{16}	V_{11}	P_5	1	C_{11}	No
S_{29}	V_{11}	P_7	1	C_{11}	No
S_{53}	V_{11}	P_9	1	C_{11}	No
S_{55_1}	V_1	P_1	0	$<1>$	Yes
S_{55_2}	V_1	P_1	0	$<1>$	Yes
S_{55_3}	V_1	P_1	0	$<1>$	Yes
S_{66}	V_1	P_1	0	$<1>$	Yes
S_{91}	V_{11}	P_3	1	C_{11}	No
S_{96}	V_1	P_1	0	$<1>$	Yes
S_{176}	V_1	P_1	0	$<1>$	Yes

Here we notice that all modules that lie in a block of defect 1 have sources that are not endo-permutation modules (except S_1). In all other cases, the simple modules lie in blocks of defect 0 and hence must have the trivial module as their source and therefore, sources are endo-permutation modules.

3.2.3 Mathieu Group M_{22}

Let us now consider the Mathieu group M_{22} with $|M_{22}| = 443520 = 2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$. It is generated by the permutations

$$x = (1, 13)((2, 8)(3, 16)(4, 12)(6, 22)(7, 17)(9, 10)(11, 14)$$

and

$$y = (1, 22, 3, 21)(2, 18, 4, 13)(5, 12)(6, 11, 7, 15)(8, 14, 20, 10)(17, 19).$$

It was originally defined by Mathieu [Cra07]. We study the sources of simple modules of M_{22} in fields of charactersitic $p \in \{2, 3, 5, 7, 11\}$. The following tables present the results of our investigation.

Characteristic 2

Table 3.9: The Group M_{22} for characteristic 2

Module	Vertex	Source	Defect	Defect Group	EP?
S_1	V_{128}	P_1	7	$(C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2) : C_2$	Yes
S_{10}	V_{128}	P_{10}	7	$(C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2) : C_2$	No
S_{10}^*	V_{128}	P_{10}	7	$(C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2) : C_2$	No
S_{34}	V_{128}	P_{34}	7	$(C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2) : C_2$	No
S_{70}	V_{128}	P_{70}	7	$(C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2) : C_2$	No
S_{70}^*	V_{128}	P_{70}	7	$(C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2) : C_2$	No
S_{98}	V_{128}	P_{98}	7	$(C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2) : C_2$	No

Here, read EP as Endo-permutation.
For M_{22} in field of characteristic 2, all the simple modules lie in the principal block. The defect of the block in 7 and vertices of the modules lie in $\mathrm{Syl}_2(M_{22})$. None of sources are endo-permutation modules.

Characteristic 3

Table 3.10: The Group M_{22} for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{21}	V_3	P_3	1	C_3	No
S_{45}	V_1	P_1	0	$< 1 >$	Yes
S_{45}^*	V_1	P_1	0	$< 1 >$	Yes
S_{49}	V_9	P_4	2	$C_3 \times C_3$	No
S_{49}^*	V_9	P_4	2	$C_3 \times C_3$	No
S_{55}	V_9	P_1	2	$C_3 \times C_3$	Yes*
S_{99}	V_1	P_1	0	$< 1 >$	Yes
S_{210}	V_3	P_3	1	C_3	No
S_{231}	V_9	P_6	2	$C_3 \times C_3$	No

Here, for modules lying in blocks of defect 1, sources are not endo-permutation modules. For modules lying in blocks of defect 2 all modules except S_{55}

sources are not endo-permutation. Except S_{55}, the only time sources are endo-permutation is when the corresponding modules lie in blocks of defect 0.

Charactersitic 5

Table 3.11: The Group M_{22} for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{21}	V_5	P_1	1	C_5	No
S_{45}	V_1	P_1	0	$< 1 >$	Yes
S_{45}^*	V_1	P_1	0	$< 1 >$	Yes
S_{55}	V_1	P_1	0	$< 1 >$	Yes
S_{98}	V_5	P_3	1	C_5	No
S_{133}	V_5	P_3	1	C_5	No
S_{210}	V_1	P_1	0	$< 1 >$	Yes
S_{280}	V_1	P_1	0	$< 1 >$	Yes
S_{280}^*	V_1	P_1	0	$< 1 >$	Yes
S_{385}	V_1	P_1	0	$< 1 >$	Yes

The modules S_{21}, S_{98}, S_{133} lie in the principal block with defect 1. Vertices of these modules lie in $\mathrm{Syl}_5(M_{22})$. sources of all these blocks are not endo-permutation modules. All other simple modules lie in blocks with defect 0 and have sources that are endo-permutation modules.

Charactersitic 7

Table 3.12: The Group M_{22} for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_{21}	V_1	P_1	0	$< 1 >$	Yes
S_{45}	V_7	P_3	1	C_7	No
S_{54}	V_7	P_5	1	C_7	No
S_{154}	V_1	P_1	0	$< 1 >$	Yes
S_{210}	V_1	P_1	0	$< 1 >$	Yes
S_{231}	V_1	P_1	0	$< 1 >$	Yes
S_{280}	V_1	P_1	0	$< 1 >$	Yes
S_{280}^*	V_1	P_1	0	$< 1 >$	Yes
S_{385}	V_1	P_1	0	$< 1 >$	Yes

The modules S_{45} and S_{54} lie in the principal block with defect 1. Vertices of these modules lie in $\mathrm{Syl}_7(M_{22})$. Sources of all these blocks are not endo-permutation modules. All other simple modules lie in blocks with defect 0 and have sources that are endo-permutation modules.

Charactersitic 11

Table 3.13: The Group M_{22} for characteristic 11

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{11}	P_1	1	C_{11}	Yes
S_{20}	V_{11}	P_9	1	C_{11}	No
S_{45_1}	V_{11}	P_1	1	C_{11}	Yes*
S_{45_2}	V_{11}	P_1	1	C_{11}	Yes*
S_{55}	V_1	P_1	0	$< 1 >$	Yes
S_{99}	V_1	P_1	0	$< 1 >$	Yes
S_{154}	V_1	P_1	0	$< 1 >$	Yes
S_{190}	V_{11}	P_1	1	C_{11}	Yes
S_{231}	V_1	P_1	0	$< 1 >$	Yes
S_{385}	V_1	P_1	0	$< 1 >$	Yes

The modules S_1, S_{20}, S_{45_1},S_{45_2}, S_{190} lie in the principal block with defect 1. Vertices of these modules lie in $\mathrm{Syl}_{11}(M_{22})$. Sources of all these modules are endo-permutation modules except S_{20}. All other simple modules lie in blocks with defect 0 and have sources that are endo-permutation modules.

3.2.4 Mathieu Group M_{23}

The Mathieu group M_{23} is of order $10200960 = 2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 23$, and is generated by the two permutations

$$x = (1,2)(3,4)(7,8)(9,10)(13,14)(15,16)(19,20)(21,22)$$

and

$$y = (1,16,11,3)(2,9,21,12)(4,5,8,23)(6,22,14,18)(13,20)(15,17).$$

This was also originally defined by Mathieu. It is characterized by being the unique simple group with centralizer of a central involution a particular extension of the elementary abelian group of order 16 by the simple group $\mathrm{PSL}_2(7)$ [Cra07]. We study this group only in fields of characteristic $p \in \{2, 3\}$. Here we depend extensively on [DK09] for our calculations. Our results are presented in tables below.

Characteristic 2

Table 3.14: The Group M_{23} for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{128}	P_1	7	$(C_4 \times C_4) : C_2) : C_2) : C_2$	Yes
S_{11}	V_{128}	$P_1 1$	7	$(C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{11}^*	V_{128}	$P_1 1$	7	$(C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{44}	V_{128}	$P_4 4$	7	$(C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{44}^*	V_{128}	$P_4 4$	7	$(C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{120}	V_{128}	P_{56}	7	$(C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{220}	V_{128}	P_{220}	7	$(C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{220}^*	V_{128}	P_{220}	7	$(C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{252}	V_{128}	P_{252}	7	$(C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{896}	V_1	P_1	0	$< 1 >$	Yes
S_{896}^*	V_1	P_1	0	$< 1 >$	Yes

The modules S_1, S_{11},S_{11}^*,S_{44},S_{44}^*,S_{120},S_{220},S_{220}^*, and S_{252} lie in the principal block with defect 1. Vertices of these modules lie in $\mathrm{Syl}_2(M_{23})$. Sources of all these modules are not endo-permutation modules (except S_1). All other simple modules lie in blocks with defect 0 and thus have sources that are endo-permutation modules.

Characteristic 3

Table 3.15: The Group M_{23} for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{22}	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{45}	V_1	P_1	0	$< 1 >$	Yes
S_{45}^*	V_1	P_1	0	$< 1 >$	Yes
S_{104}	V_9	P_5	2	$C_3 \times C_3$	No
S_{104}	V_9	P_5	2	$C_3 \times C_3$	No
S_{231}	V_3	P_1	1	C_3	Yes
S_{253}	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{770}	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{770}^*	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{990}	V_1	P_1	0	$< 1 >$	Yes
S_{990}^*	V_1	P_1	0	$< 1 >$	Yes
S_{1035}	V_1	P_1	0	$< 1 >$	Yes

The modules S_1, S_{22},S_{104},S_{104}^*,S_{253},S_{770},S_{770}^* and S_{220}^* lie in the principal block with defect 1. Vertices of these modules lie in $\mathrm{Syl}_3(M_{23})$. Sources of all these modules are endo-permutation modules, with two exceptions. Module S_{231} lies in block with defect 1, and all other simple modules lie in blocks with defect 0. In all these cases modules have sources that are endo-permutation modules.

3.3 Janko Groups

We investigate the Janko groups J_1 and J_2 and present our results.

3.3.1 Janko Group J_1

The first Janko group has order $175560 = 2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 19$. The group has presentation

$$< a, b \colon a^2 = b^3 = (ab)^7 = (ab(abab^{-1})^3)^5 = (ab(abab^{-1})^6 abab(ab^{-1})^2)^2 = 1 >$$

Although it has a (relatively) easy permutation representation on 266 points, it can also be represented as 20-dimensional matrices over GF(2), and as 7-dimensional matrices over GF(11). It was first considered by Janko, and is the only sporadic simple group with abelian Sylow 2-subgroups[Cra07]. We study simple modules of $J-1$ in fields of charactersistic $p \in \{2, 3, 5, 7, 11, 19\}$

Characteristic 2

Table 3.16: The Group J_1 for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_8	P_1	3	$C_2 \times C_2 \times C_2 \times C_2$	Yes
S_{20}	V_8	$P_1 2$	3	$C_2 \times C_2 \times C_2 \times C_2$	No
S_{56_1}	V_1	P_1	0	$< 1 >$	Yes
S_{56_2}	V_1	P_1	0	$< 1 >$	Yes
S_{56_3}	V_8	P_8	3	$C_2 \times C_2 \times C_2 \times C_2$	No
S_{56_4}	V_8	P_8	3	$C_2 \times C_2 \times C_2 \times C_2$	No
S_{76_1}	V_2	P_1	1	C_2	Yes
S_{76_2}	V_8	P_{12}	3	$C_2 \times C_2 \times C_2 \times C_2$	No
S_{120_1}	V_1	P_1	0	$< 1 >$	Yes
S_{120_2}	V_1	P_1	0	$< 1 >$	Yes
S_{120_3}	V_1	P_1	0	$< 1 >$	Yes

The modules S_1, S_{20},S_{56_3},S_{56_4} and S_{76_2} lie in the principal block with defect 3. Vertices of these modules lie in $\mathrm{Syl}_2(M_{23})$. Sources of all these modules are not endo-permutation modules (except S_1). Module S_{76_1} lies in block with defect 1 and all other simple modules lie in blocks with defect 0 and have sources that are endo-permutation modules.

Characteristic 3

Table 3.17: The Group J_1 for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{56}	V_3	P_1	1	C_3	Yes*
S_{56}	V_3	P_1	1	C_3	Yes*
S_{76}	V_3	P_1	1	C_3	Yes*
S_{76}	V_3	P_1	1	C_3	Yes*
S_{77}	V_3	P_1	1	C_3	Yes*
S_{77}	V_3	P_1	1	C_3	Yes*
S_{120}	V_1	P_1	0	$< 1 >$	Yes
S_{120}	V_1	P_1	0	$< 1 >$	Yes
S_{120}	V_1	P_1	0	$< 1 >$	Yes
S_{133}	V_3	P_1	1	C_3	Yes*

The simple modules of group J_1 in field of charaacteristic 3 is an example where all the simple modules have sources that are endo-permutation modules.

Characteristic 5

Table 3.18: The Group J_1 for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{56}	V_5	P_3	1	C_5	No
S_{76_1}	V_5	P_1	1	C_5	Yes*
S_{76_2}	V_5	P_1	1	C_5	Yes*
S_{77}	V_5	P_1	1	C_5	Yes*
S_{120_1}	V_1	P_1	0	$< 1 >$	Yes
S_{120_2}	V_1	P_1	0	$< 1 >$	Yes
S_{120_3}	V_1	P_1	0	$< 1 >$	Yes
S_{133}	V_5	P_3	1	C_5	No

Here, we have only two cases; S_{56} and S_{133}; with sources that are not endo-permutation modules. Both these modules lie in block of defect 1 with vertices in $\mathrm{Syl}_5(J_1)$. Modules S_{76_1}, S_{76_2} and S_{77} also lie in block of defect 1 but have trivial sources. These 3 modules are also examples of the fact that converse of theorem 2.5.1 is not true. All other modules lie in blocks of defect 0 and hence have trivial module as source.

Characteristic 7

Table 3.19: The Group J_1 for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_{31}	V_7	P_3	1	C_7	No
S_{45}	V_7	P_3	1	C_7	No
S_{56}	V_1	P_1	0	$< 1 >$	Yes
S_{56}	V_1	P_1	0	$< 1 >$	Yes
S_{75}	V_7	P_5	1	C_7	No
S_{77}	V_1	P_1	0	$< 1 >$	Yes
S_{77}	V_1	P_1	0	$< 1 >$	Yes
S_{77}	V_1	P_1	0	$< 1 >$	Yes
S_{89}	V_7	P_5	1	C_7	No
S_{120}	V_7	P_1	1	C_7	Yes*
S_{133}	V_1	P_1	0	$< 1 >$	Yes
S_{133}	V_1	P_1	0	$< 1 >$	Yes
S_{133}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_{120} all the simple modules in block of defect 1 have sources that are not endo-permutaiton modules. All other modules lie in blocks of defect 0 and have trivial module as source. Hence sources are endo-permutation moules.

Characteristic 11

Table 3.20: The Group J_1 for characteristic 11

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{11}	P_1	1	C_{11}	Yes
S_7	V_{11}	P_7	1	C_{11}	No
S_{14}	V_{11}	P_3	1	C_{11}	No
S_{27}	V_{11}	P_5	1	C_{11}	No
S_{49}	V_{11}	P_5	1	C_{11}	No
S_{56}	V_{11}	P_1	1	C_{11}	Yes*
S_{64}	V_{11}	S_9	1	C_{11}	No
S_{69}	V_{11}	P_3	1	C_{11}	No
S_{77_1}	V_1	P_1	0	$< 1 >$	Yes
S_{77_2}	V_1	P_1	0	$< 1 >$	Yes
S_{77_3}	V_1	P_1	0	$< 1 >$	Yes
S_{106}	V_{11}	P_7	1	C_{11}	No
S_{119}	V_{11}	P_9	1	C_{11}	No
S_{209}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_{56} all the simple modules in block of defect 1; S_7, S_{14}, S_{27}, S_{49}, S_{64}, S_{69}, S_{106}, S_{119}; have sources that are not endo-permutaiton modules. All other modules lie in blocks of defect 0 and have trivial module as source, and hence endo-permutation.

Characteristic 19

Table 3.21: The Group J_1 for characteristic 19

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{19}	P_1	1	C_{19}	Yes
S_{22}	V_{19}	P_3	1	C_{19}	No
S_{34}	V_{19}	P_{15}	1	C_{19}	No
S_{43}	V_{19}	P_5	1	C_{19}	No
S_{55}	V_{19}	P_{17}	1	C_{19}	No
S_{76}	V_1	P_1	0	$< 1 >$	Yes
S_{76}	V_1	P_1	0	$< 1 >$	Yes
S_{77}	V_{19}	P_1	1	C_{19}	Yes*
S_{133}	V_1	P_1	0	$< 1 >$	Yes
S_{133}	V_1	P_1	0	$< 1 >$	Yes
S_{133}	V_1	P_1	0	$< 1 >$	Yes
S_{209}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_{77} all the simple modules in block of defect 1; S_{22}, S_{34}, S_{43}, S_{55}; have sources that are not endo-permutaiton modules. Vertices in these cases lie in $\mathrm{Syl}_19(J_1)$. All other modules lie in blocks of defect 0 and have trivial module as sources. Hence sources are endo-permutation modules.

3.3.2 Janko Group J_2

The second Janko group J_2 is of order $604800 = 2^7 \cdot 3^3 \cdot 5^2 \cdot 7$. The group has presentation

$$< a, b | a^2 = b^3 = (ab)^7 = [a, b]^{12} = (abababab^{-1}abab^{-1}ab^{-1}abababab^{-1}ab^{-1}abab^{-1})^3 = 1 >$$

[Con+85].
We study the simple modules in fields of characteristic $p \in \{2, 3, 5, 7\}$

Characteristic 2

Table 3.22: The Group J_2 for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{128}	P_1	7	$(((C_4 \times C_4) : C_2) : C_2) : C_2$	Yes
S_{6_1}	V_{128}	P_6	7	$(((C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{6_2}	V_{128}	P_6	7	$(((C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{14_1}	V_{128}	P_{14}	7	$(((C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{14_2}	V_{128}	P_{14}	7	$(((C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{36}	V_{128}	P_{36}	7	$(((C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{64_1}	V_4	P_2	2	$C_2 \times C_2$	No
S_{64_2}	V_4	P_2	2	$C_2 \times C_2$	No
S_{84}	V_{128}	P_{84}	7	$(((C_4 \times C_4) : C_2) : C_2) : C_2$	No
S_{160}	V_4	P_1	2	$C_2 \times C_2$	Yes*

With the exception of S_{160} all the simple modules; S_{6_1}, S_{6_2}, S_{14_1}, S_{14_2}, S_{36}, S_{64_1}, S_{74_2}, S_{84}; have sources that are not endo-permutaiton modules. There are no modules in a block of defect 0.

Characteristic 3

Table 3.23: The Group J_2 for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{27}	P_1	3	$(C_3 \times C_3) : C_3$	Yes
S_{13_1}	V_{27}	P_{13}	3	$(C_3 \times C_3) : C_3$	No
S_{13_2}	V_{27}	P_{13}	3	$(C_3 \times C_3) : C_3$	No
S_{21_1}	V_{27}	P_{13}	3	$(C_3 \times C_3) : C_3$	No
S_{21_2}	V_{27}	P_{21}	3	$(C_3 \times C_3) : C_3$	No
S_{36}	V_3	P_1	1	C_3	Yes*
S_{57_1}	V_{27}	P_3	3	$(C_3 \times C_3) : C_3$	No
S_{57_2}	V_{27}	P_3	3	$(C_3 \times C_3) : C_3$	No
S_{63}	V_3	P_1	1	C_3	Yes*
S_{90}	V_3	P_1	1	C_3	Yes*
S_{133}	V_{27}	P_{79}	3	$(C_3 \times C_3) : C_3$	No
S_{189_1}	V_1	P_1	0	$< 1 >$	Yes
S_{189_2}	V_1	P_1	0	$< 1 >$	Yes
S_{225}	V_3	P_1	1	C_3	Yes*

Here, modules in the principal block; S_{13_1}, S_{13_2},S_{21_1}, S_{21_2} S_{57_1}, S_{57_2},S_{133}; lie in block of defect 3 with vertices lying in $\mathrm{Syl}_3(J_2)$ and have sources that are not endo-permutation modules. In all other cases, sources are the trivial module and hence endo-permutation. In cases of modules in blocks of defect 1, these serve to show that the converse of theorem 2.5.1 is not true.

Characteristic 5

Table 3.24: The Group J_2 for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{25}	P_1	2	$C_5 \times C_5$	Yes
S_{14}	V_{25}	P_{14}	2	$C_5 \times C_5$	No
S_{21}	V_{25}	P_{21}	2	$C_5 \times C_5$	No
S_{41}	V_{25}	P_{16}	2	$C_5 \times C_5$	No
S_{70}	V_5	P_3	1	C_5	No
S_{85}	V_{25}	P_{10}	2	$C_5 \times C_5$	No
S_{90}	V_5	P_1	1	C_5	Yes*
S_{175}	V_1	P_1	0	$< 1 >$	Yes
S_{189}	V_{25}	P_1	2	$C_5 \times C_5$	No
S_{225}	V_1	P_1	0	$< 1 >$	Yes
S_{300}	V_1	P_1	0	$< 1 >$	Yes

Here, modules in the principal block; S_{14}, S_{21},S_{41}, S_{85} S_{175}, S_{57_2}; lie in block of defect 2 with vertices lying in $\mathrm{Syl}_5(J_2)$ and have sources that are not endo-permutation modules. In all other cases, sources are the trivial module and hence endo-permutation. In cases of modules in blocks of defect 1, these serve to show that the converse of theorem 2.5.1 is not true.

Characteristic 7

Table 3.25: The Group J_2 for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_5	Yes
S_{14_1}	V_1	P_1	0	$< 1 >$	Yes
S_{14_2}	V_1	P_1	0	$< 1 >$	Yes
S_{21_1}	V_1	P_1	0	$< 1 >$	Yes
S_{21_2}	V_1	P_1	0	$< 1 >$	Yes
S_{36}	V_7	P_1	1	C_5	Yes*
S_{63}	V_1	P_1	0	$< 1 >$	Yes
S_{70_1}	V_1	P_1	0	$< 1 >$	Yes
S_{70_2}	V_1	P_1	0	$< 1 >$	Yes
S_{89}	V_7	P_5	1	C_5	No
S_{101}	V_7	P_3	1	C_5	No
S_{124}	V_7	P_5	1	C_5	No
S_{126}	V_1	P_1	0	$< 1 >$	Yes
S_{175}	V_1	P_1	0	$< 1 >$	Yes
S_{189_1}	V_1	P_1	0	$< 1 >$	Yes
S_{189_2}	V_1	P_1	0	$< 1 >$	Yes
S_{199}	V_7	S_3	1	C_5	No
S_{224_1}	V_1	P_1	0	$< 1 >$	Yes
S_{224_2}	V_1	P_1	0	$< 1 >$	Yes
S_{336}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_{36} all the simple modules in block of defect 1; S_{89}, S_{101}, S_{124}, S_{199}; have sources that are not endo-permutaiton modules. Vertices in these cases lie in $\mathrm{Syl}_2(J_2)$. All other modules lie in blocks of defect 0 and have trivial module as sources. Hence sources are endo-permutation modules.

Chapter 4

Simple Alternating Groups

4.1 The Alternating Group A_5

The alternating group $A_5 \cong L_2(4) \cong L_2(5)$ is the smallest alternating group of order $60 = 2^2 \cdot 3 \cdot 5$. It is generated by the permutations $x = (1,2)(3,4)$ and $y = (1,3,5)$. It's standard presentation is given by

$$< a, b | a^2 = b^3 = (ab)^5 = 1 >$$

[Con+85]. We study the modules of this group in fields of characteristic $p \in \{2,3,5\}$ and present our results below.

4.1.1 Characteristic 2

Table 4.1: The Group A_5 for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_4	P_1	2	$C_2 \times C_2$	Yes
S_{2_1}	V_4	P_2	2	$C_2 \times C_2$	No
S_{2_2}	V_4	P_2	2	$C_2 \times C_2$	No
S_4	V_1	P_1	0	$< 1 >$	Yes

All the simple modules in block of defect 2; S_{2_1}, S_{2_2}; have sources that are not endo-permutaiton modules. Vertices in these cases lie in $\mathrm{Syl}_2(A_5)$. All other modules lie in blocks of defect 0 and have trivial module as sources. Hence, these sources are endo-permutation modules.

4.1.2 Characteristic 3

Table 4.2: The Group A_5 for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_3	V_1	P_1	0	$< 1 >$	Yes
S_3	V_1	P_1	0	$< 1 >$	Yes
S_4	V_3	P_1	1	C_3	Yes

Here we have that all simple modules have as source the trivial 1 dimensional module, and are hence endo-permutation.

4.1.3 Characteristic 5

Table 4.3: The Group A_5 for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_3	V_5	P_3	1	C_5	No
S_5	V_1	P_1	0	$< 1 >$	Yes

Module S_3 lies in block of defect 1 has sources that is not endo-permutation. Vertices of modules in block of defect 1 lie in $\mathrm{Syl}_5(A_5)$.

4.2 The Alternating Group A_6

The group $A_6 \cong L_2(9)$ has order $360 = 2^3.3^2.5$. It is generated by permutations $x = (1, 2)(3, 4)$ and $y = !1, 2, 3, 5)(4, 6)$. It's presentation is given by

$$< a, b | a^2 = b^4 = (ab)^5 = (ab^2)^5 = 1 >$$

[Con+85]. We study simple modules of this group in fields of characteristic $p \in \{2, 3, 5\}$ and present our results below.

4.2.1 Characteristic 2

Table 4.4: The Group A_6 for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_8	P_1	3	D_8	Yes
S_{4_1}	$C_2 \times C_2$	P_4	3	D_8	No
S_{4-2}	$C_2 \times C_2$	P_4	3	D_8	No
S_8	V_1	P_1	0	$< 1 >$	Yes
S_9	V_1	P_1	0	$< 1 >$	Yes

All the simple modules in block of defect 3; S_{4_1}, S_{4_2}; have sources that are not endo-permutaiton modules. Vertices in these cases lie in $\mathrm{Syl}_2(A_6)$. All other modules lie in blocks of defect 0 and have trivial module as sources. Hence, these sources are endo-permutation modules.

4.2.2 Characteristic 3

Table 4.5: The Group A_6 for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_9	P_1	3	$C_3 \times C_3$	Yes
S_{3_1}	V_9	P_3	3	$C_3 \times C_3$	No
S_{3_2}	V_9	P_3	3	$C_3 \times C_3$	No
S_4	V_9	P_3	3	$C_3 \times C_3$	No
S_9	V_1	P_1	0	$< 1 >$	Yes

All the simple modules in block of defect 2; S_{3_1}, S_{3_2}, S_4; have sources that are not endo-permutaiton modules. Vertices in these cases lie in $\mathrm{Syl}_2(A_5)$. All other modules lie in blocks of defect 0 and have trivial module as sources. Hence, these sources are endo-permutation modules.

4.3 The Alternating Group A_7

The alternating group A_7 has order $2520 = 2^3 \cdot 3^2 \cdot 5 \cdot 7$. It is generated by the permutation $x = (1, 2, 3)$ and $y = (3, 4, 5, 6, 7)$. It has as it's standard presentation

$$< a, b | a^3 = b^5 = (ab)^7 = (aab)^2 = (ab^{-2}ab^2)^2 = 1 > .$$

We study the simple modules in fields of characteristic $p \in \{2, 3, 5, 7\}$ and present our results below [Con+85].

4.3.1 Characteristic 2

Table 4.6: The Group A_7 for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_8	P_1	3	D_8	Yes
S_{4_1}	V_4	P_4	2	$C_2 \times C_2$	No
S_{4_2}	V_4	P_4	2	$C_2 \times C_2$	No
S_6	V_4	P_1	2	$C_2 \times C_2$	Yes
S_{14}	V_4	P_1	2	$C_2 \times C_2$	Yes
S_{20}	V_4	P_4	2	$C_2 \times C_2$	No

All the simple modules in block of defect 2. Vertices in these cases lie in $\mathrm{Syl}_2(A_7)$. The modules S_{4_1}, S_{4_2} and S_{20} have sources that are not endo-permutation modules. All other modules have sources that is the trivial 1 dimensional module and are hence endo-permutation.

4.3.2 Characteristic 3

Table 4.7: The Group A_7 for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_9	P_1	2	$C_3 \times C_3$	Yes
S_6	V_3	P_1	1	C_3	Yes
S_{10_1}	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{10_2}	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{13}	V_9	P_4	2	$C_3 \times C_3$	No
S_{15}	V_3	P_1	1	C_3	Yes

With the exception of S_{13}, all simple modules have sources that are endo-permutaiton modules.

4.3.3 Characteristic 5

Table 4.8: The Group A_7 for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_6	V_5	P_1	1	C_5	Yes
S_8	V_5	P_3	1	C_5	No
S_{10_1}	V_1	P_1	0	$< 1 >$	Yes
S_{10_2}	V_1	P_1	0	$< 1 >$	Yes
S_{13}	V_5	P_3	1	C_5	No
S_{15}	V_1	P_1	0	$< 1 >$	Yes
S_{35}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_6, all simple modules in blocks of defect 1 have sources that are not endo-permutaiton modules.Vertices lie in $\mathrm{Syl}_5(A_7)$. All other modules lie in blocks of defect 0 and therefore have sources that endo-permutation modules.

4.3.4 Characteristic 7

Table 4.9: The Group A_7 for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_5	V_7	P_5	1	C_7	No
S_{10}	V_7	P_3	1	C_7	No
S_{14_1}	V_1	P_1	0	$< 1 >$	Yes
S_{14_1}	V_1	P_1	0	$< 1 >$	Yes
S_{21}	V_1	P_1	1	$< 1 >$	Yes
S_{35}	V_1	P_1	0	$< 1 >$	Yes

All simple modules in blocks of defect 1 have sources that are not endo-permutaiton modules.Vertices lie in $\mathrm{Syl}_7(A_7)$. All other modules lie in blocks of defect 0 and therefore have sources that endo-permutation modules.

4.4 The Alternating Group A_8

The alternating group $A_8 \cong L_4(2)$ has order $20160 = 2^6 \cdot 3^2 \cdot 5 \cdot 7$. It is generated by the permutations $x = (1, 2, 3)$ and $y = (2, 3, 4, 5, 6, 7, 8)$. We study simple modules in fields of characteristic $p \in \{2, 3, 5, 7\}$ and present oour results below [Con+85].

4.4.1 Characteristic 2

Table 4.10: The Group A_8 for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{64}	P_1	6	$(C_2 \times C_2 \times C_2 \times C_2) : C_2 : C_2$	Yes
S_4	V_{64}	P_4	6	$(C_2 \times C_2 \times C_2 \times C_2) : C_2 : C_2$	No
S_4	V_{64}	P_4	6	$(C_2 \times C_2 \times C_2 \times C_2) : C_2 : C_2$	No
S_6	V_{64}	P_6	6	$(C_2 \times C_2 \times C_2 \times C_2) : C_2 : C_2$	No
S_{14}	V_{64}	P_{14}	6	$(C_2 \times C_2 \times C_2 \times C_2) : C_2 : C_2$	No
S_{20}	V_{64}	P_{20}	6	$(C_2 \times C_2 \times C_2 \times C_2) : C_2 : C_2$	No
S_{20}	V_{64}	P_{20}	6	$(C_2 \times C_2 \times C_2 \times C_2) : C_2 : C_2$	No
S_{64}	V_1	P_1	0	$< 1 >$	Yes

All simple modules in blocks of defect 6 have sources that are not endo-permutaiton modules.Vertices lie in $\mathrm{Syl}_2(A_8)$. All other modules lie in blocks of defect 0 and therefore have sources that endo-permutation modules.

4.4.2 Characteristic 3

Table 4.11: The Group A_8 for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_9	P_1	2	$C_3 \times C_3$	Yes
S_7	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{13}	V_9	P_4	2	$C_3 \times C_3$	No
S_{21}	V_3	P_3	1	C_3	No
S_{28}	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{35}	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{45_1}	V_1	P_1	0	$< 1 >$	Yes
S_{45_2}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_{13}, all simple modules in blocks of defect 1 have sources that are endo-permutaiton modules.Vertices lie in $\mathrm{Syl}_3(A_8)$. Module S_{21} lies in block of defect 1 and has source that is not endo-permutation. All other modules lie in blocks of defect 0 and therefore have sources that endo-permutation modules.

4.4.3 Characteristic 5

Table 4.12: The Group A_8 for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_7	V_5	P_1	1	C_5	Yes
S_{13}	V_5	P_3	1	C_5	No
S_{20}	V_1	P_1	0	$< 1 >$	Yes
S_{21_1}	V_5	P_1	1	C_5	Yes
S_{21_2}	V_5	P_3	1	C_5	No
S_{35}	V_1	P_1	0	$< 1 >$	Yes
S_{43}	V_5	P_3	1	C_5	No
S_{45_1}	V_1	P_1	0	$< 1 >$	Yes
S_{45_2}	V_1	P_1	0	$< 1 >$	Yes
S_{70}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_7, all simple modules in blocks of defect 1 have sources that are not endo-permutaiton modules.Vertices lie in $\mathrm{Syl}_5(A_8)$. All other modules lie in blocks of defect 0 and therefore have sources that endo-permutation modules.

4.4.4 Characteristic 7

Table 4.13: The Group A_8 for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_7	V_1	P_1	0	$< 1 >$	Yes
S_{14}	V_1	P_1	0	$< 1 >$	Yes
S_{19}	V_7	P_5	1	C_7	No
S_{21}	V_1	P_1	0	$< 1 >$	Yes
S_{21}	V_1	P_1	0	$< 1 >$	Yes
S_{21}	V_1	P_1	0	$< 1 >$	Yes
S_{28}	V_1	P_1	0	$< 1 >$	Yes
S_{35}	V_1	P_1	0	$< 1 >$	Yes
S_{45}	V_7	P_3	1	C_7	No
S_{56}	V_1	P_1	0	$< 1 >$	Yes
S_{70}	V_1	P_1	0	$< 1 >$	Yes

The modules S_{14} and S_{45} lie in the principal block of defect 1. Vertices of these modules lie in $\mathrm{Syl}_7(A_8)$ and sources are not endo-permutation modules. All other modules lie in blocks of defect 0 and hence have sources that are endo-permutation modules.

4.5 The Alternating Group A_9

The alternating group A_9 has order $181440 = 2^6 \cdot 3^4 \cdot 5 \cdot 7$. It can be generated by the permutations $x = (1, 2, 3)$ and $y = (3, 4, 5, 6, 7, 8, 9)$ [Con+85]. We study the simple modules in fields of characteristic $p \in \{2, 3, 5, 7\}$ and present our results below.

4.5.1 Characteristic 2

Table 4.14: The Group A_9 for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{64}	P_1	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	Yes
S_8	V_8	P_2	3	D_8	Yes
S_8	V_{64}	P_8	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_8	V_{64}	P_8	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{20}	V_{64}	P_{20}	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{20}	V_{64}	P_{20}	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{26}	V_{64}	P_{26}	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{48}	$C_2 \times C_2$	P_1	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	Yes*
S_{78}	V_{64}	P_{14}	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{160}	$C_2 \times C_2$	P_4	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No

All simple modules in blocks of defect 4 have sources that are not endo-permutaiton modules. Vertices lie in $\mathrm{Syl}_2(A_9)$. All other modules lie in blocks of defect 0 and therefore have sources that endo-permutation modules.

4.5.2 Charecteristic 3

Table 4.15: The Group A_9 for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{81}	P_1	4	$(C_3 \times C_3 \times C_3) : C_3$	Yes
S_7	V_{81}	P_7	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{21}	V_{81}	P_{21}	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{27}	V_3	P_1	1	C_3	Yes*
S_{35}	V_{81}	P_{35}	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{41}	V_{81}	P_{41}	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{162}	V_1	P_1	0	$< 1 >$	Yes
S_{189}	V_3	P_1	1	C_3	Yes*

All simple modules in blocks of defect 4 have sources that are not endo-permutaiton modules. Vertices lie in $\mathrm{Syl}_3(A_9)$. All modules that lie in blocks of defect 1 and defect 0 have sources that endo-permutation modules.

4.5.3 Characteristic 5

Table 4.16: The Group A_9 for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_8	V_5	P_1	1	C_5	Yes
S_{21}	V_5	P_3	1	C_5	No
S_{27}	V_5	P_1	1	C_5	Yes*
S_{28}	V_5	P_1	1	C_5	Yes*
S_{34}	V_5	P_3	1	C_5	No
S_{35}	V_1	P_1	0	$< 1 >$	Yes
S_{35}	V_1	P_1	0	$< 1 >$	Yes
S_{56}	V_5	P_1	1	C_5	Yes*
S_{83}	V_5	P_3	1	C_5	No
S_{105}	V_1	P_1	0	$< 1 >$	Yes
S_{120}	V_1	P_1	0	$< 1 >$	Yes
S_{133}	V_5	P_3	1	C_5	No
S_{134}	V_5	P_3	1	C_5	No

4.5.4 Charactersitic 7

Table 4.17: The Group A_9 for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_8	V_7	P_1	1	C_7	Yes*
S_{19}	V_7	P_5	1	C_7	No
S_{21_1}	V_1	P_1	0	$< 1 >$	Yes
S_{21_2}	V_1	P_1	0	$< 1 >$	Yes
S_{28}	V_1	P_1	0	$< 1 >$	Yes
S_{35}	V_1	P_1	0	$< 1 >$	Yes
S_{35}	V_1	P_1	0	$< 1 >$	Yes
S_{42}	V_1	P_1	0	$< 1 >$	Yes
S_{47}	V_7	P_5	1	C_7	No
S_{56}	V_1	P_1	0	C_7	Yes
S_{84}	V_1	P_1	0	C_7	Yes
S_{101}	V_7	P_3	1	C_7	No
S_{105}	V_1	P_1	0	$< 1 >$	Yes
S_{115}	V_7	P_3	1	C_7	No
S_{168}	V_1	P_1	0	$< 1 >$	Yes
S_{189}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_8 all simple modules in blocks of defect 1 have sources that are not endo-permutaiton modules. Vertices lie in $\mathrm{Syl}_7(A_9)$. All modules that lie in blocks of defect defect 0 have sources that endo-permutation modules.

4.6 The Alternating Group A_{10}

The alternating group A_{10} has order $1814400 = 2^7 \cdot 3^4 \cdot 5^2 \cdot 7$ and can be generated by the permutations $x = (1, 2, 3)$ and $y = (2, 3, 4, 5, 6, 7, 8, 9, 10)$. We study idecomposable modules in fields of charactersic $p \in \{2, 3, 5, 7\}$ and present our results below[Con+85].

Characteristic 2

Table 4.18: The Group A_{10} for characteristic 2

Module	Vertex	Source	Defect	Defect Group	EP?
S_1	V_{128}	P_1	7	$(D_8 \times D_8) : C_2$	Yes
S_8	V_{128}	P_8	7	$(D_8 \times D_8) : C_2$	No
S_{16}	V_{64}	P_8	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{26}	V_{128}	P_{26}	7	$(D_8 \times D_8) : C_2$	No
S_{48}	V_{128}	P_{48}	7	$(D_8 \times D_8) : C_2$	No
S_{64}	V_4	P_4	2	$C_2 \times C_2$	No
S_{64}	V_4	P_{26}	2	$C_2 \times C_2$	No
S_{160}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{198}	V_{128}	P_{70}	7	$(D_8 \times D_8) : C_2$	No
S_{200}	V_{64}	P_{20}	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{384}	V_1	P_1	0	$< 1 >$	Yes
S_{384}	V_1	P_1	0	$< 1 >$	Yes

Read EP as Endo-permutation.
With the exception of S_{160} and modules lying in blocks with defect 0; all simple modules in blocks of have sources that are not endo-permutaiton modules.Vertices lie in $\mathrm{Syl}_2(A_{10})$.

4.6.1 Characteristic 3

Table 4.19: The Group A_{10} for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{81}	P_1	4	$(C_3 \times C_3 \times C_3) : C_3$	Yes
S_9	V_9	P_1	2	$(C_3 \times C_3$	Yes
S_{34}	V_{81}	P_7	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{36}	V_9	P_1	2	$(C_3 \times C_3$	Yes*
S_{41}	V_{81}	P_{41}	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{84}	V_{27}	P_1	4	$C_3 \times C_3 \times C_3$	Yes*
S_{90}	V_9	P_1	2	$(C_3 \times C_3$	Yes*
S_{126}	V_9	P_1	2	$(C_3 \times C_3$	Yes*
S_{224}	V_{81}	P_{35}	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{279}	V_9	P_4	2	$(C_3 \times C_3$	No
S_{567}	V_1	P_1	0	$< 1 >$	Yes

All the simple modules that lie in the principal block with defect 4 ; S_9, S_{34}, S_{41}, and S_{224} ; have sources that are not endo-permutation modules. Vertices of these modules lie in $\mathrm{Syl}_3(A_{10})$. All other modules have sources that are endo-permutaiton modules.

4.6.2 Charactersitic 5

Table 4.20: The Group A_{10} for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{25}	P_1	2	$C_5 \times C_5$	Yes
S_8	V_{25}	P_8	2	$C_5 \times C_5$	No
S_{28}	V_{25}	P_{28}	2	$C_5 \times C_5$	No
S_{34}	V_{25}	P_9	2	$C_5 \times C_5$	No
S_{35}	V_5	P_9	1	C_5	Yes
S_{35}	V_{25}	P_{10}	2	$C_5 \times C_5$	No
S_{35}	V_{25}	P_{10}	2	$C_5 \times C_5$	No
S_{55}	V_5	P_5	1	C_5	No
S_{56}	V_{25}	P_{56}	2	$C_5 \times C_5$	No
S_{75}	V_1	P_1	0	$< 1 >$	Yes
S_{133}	V_{25}	P_8	2	$C_5 \times C_5$	No
S_{133}	V_{25}	P_8	2	$C_5 \times C_5$	No
S_{155}	V_5	P_3	1	C_5	No
S_{160}	V_5	P_1	1	C_5	Yes*
S_{217}	V_{25}	P_{42}	2	$C_5 \times C_5$	No
S_{225}	V_1	P_1	0	$< 1 >$	Yes
S_{300}	V_1	P_1	0	$< 1 >$	Yes
S_{350}	V_1	P_1	0	$< 1 >$	Yes
S_{450}	V_1	P_1	0	$< 1 >$	Yes
S_{525}	V_1	P_1	0	$< 1 >$	Yes

All simple modules in blocks of defect 2 have sources that are not endo-permutaiton modules.Vertices lie in $\mathrm{Syl}_5(A_{10})$. All modules that lie in blocks of defect defect 0 have sources that endo-permutation modules.

4.6.3 Characteristic 7

Table 4.21: The Group A_{10} for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_9	V_7	P_1	1	C_7	Yes*
S_{35}	V_1	P_{35}	0	$< 1 >$	Yes
S_{36}	V_7	P_1	1	C_7	Yes*
S_{42}	V_1	P_{42}	0	$< 1 >$	Yes
S_{66}	V_7	P_5	1	C_7	No
S_{84}	V_1	P_{84}	0	$< 1 >$	Yes
S_{89}	V_7	P_5	1	C_7	No
S_{101}	V_7	P_3	1	C_7	No
S_{124}	V_7	P_5	1	C_7	No
S_{126}	V_1	P_{126}	0	$< 1 >$	Yes
S_{199}	V_7	P_3	1	C_7	No
S_{210}	V_1	P_{210}	0	$< 1 >$	Yes
S_{224}	V_1	P_{224}	0	$< 1 >$	Yes
S_{224}	V_1	P_{224}	0	$< 1 >$	Yes
S_{252}	V_1	P_{252}	0	$< 1 >$	Yes
S_{315}	V_1	P_{315}	0	$< 1 >$	Yes
S_{350}	V_1	P_{350}	0	$< 1 >$	Yes
S_{384}	V_7	P_3	1	C_7	No
S_{525}	V_1	P_{525}	0	$< 1 >$	Yes
S_{567}	V_1	P_{567}	0	$< 1 >$	Yes

Chapter 5

Simple Linear Groups

5.1 The Linear Group $L_2(7)$

Linear group $L_2(7) \cong L_3(2)$ is of order $168 = 2^3 \cdot 3 \cdot 7$ It has standard presentation $< a, b | a^2 = b^3 = (ab)^7 = [a, b]^4 = 1 >$. We study simple modules in fields of characteristic $p \in \{2, 3, 7\}$ and present our results below.

5.1.1 Characteristic 2

Table 5.1: The Group $L_2(7)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_8	P_1	3	D_8	Yes
S_3	V_8	P_3	3	D_8	No
S_3	V_8	P_3	3	D_8	No
S_8	V_1	P_1	0	$< 1 >$	Yes

All simple modules in blocks of defect 2 have sources that are not endo-permutation modules.Vertices lie in $\mathrm{Syl}_2(L_27)$. All modules that lie in blocks of defect defect 0 have sources that are endo-permutation modules.

5.1.2 Characteristic 3

Table 5.2: The Group $L_2(7)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_3	V_1	P_1	0	$< 1 >$	Yes
S_3	V_1	P_1	0	$< 1 >$	Yes
S_6	V_1	P_1	0	$< 1 >$	Yes
S_7	V_3	P_1	1	C_3	Yes*

In this case, all simple modules have sources that are endo-permutation modules.

5.1.3 Characteristic 7

Table 5.3: The Group $L_2(7)$ for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_3	V_7	P_3	1	C_7	No
S_5	V_7	P_5	1	C_7	No
S_7	V_1	P_1	0	$< 1 >$	Yes

All simple modules in blocks of defect 1 have sources that are not endo-permutation modules. Vertices lie in $\mathrm{Syl}_7(L2_7)$. All modules that lie in blocks of defect defect 0 have sources that endo-permutation modules.

5.2 The Linear Group $L_2(8)$

Linear group $L_2(7) \cong L_3(2)$ is of order $504 = 2^3 \cdot 3^2 \cdot 7$ It has standard presentation $< a, b | a^2 = b^3 = (ab)^7 = (ababab^{-1}ababab^{-1}ab^{-1})^2 = 1 >$. We study simple modules in fields of characteristic $p \in \{2, 3, 7\}$ and present our results below.

5.2.1 Characteristic 2

Table 5.4: The Group $L_2(8)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_8	P_1	3	$C_3 \times C_3 \times C_3$	Yes
S_2	V_8	P_2	3	$C_3 \times C_3 \times C_3$	No
S_2	V_8	P_3	3	$C_3 \times C_3 \times C_3$	No
S_2	V_8	P_2	3	$C_3 \times C_3 \times C_3$	No
S_4	V_8	P_4	3	$C_3 \times C_3 \times C_3$	No
S_4	V_8	P_4	3	$C_3 \times C_3 \times C_3$	No
S_4	V_8	P_4	3	$C_3 \times C_3 \times C_3$	No
S_8	V_1	P_1	0	$< 1 >$	Yes

All simple modules in blocks of defect 3 have sources that are not endo-permutation modules. Vertices lie in $\mathrm{Syl}_2(L2_8)$.

5.2.2 Characteristic 3

Table 5.5: The Group $L_2(8)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_9	P_1	2	C_9	Yes
S_7	V_9	P_7	2	C_9	No
S_9	V_1	P_1	0	$< 1 >$	Yes
S_9	V_1	P_1	0	$< 1 >$	Yes
S_9	V_1	P_1	0	$< 1 >$	Yes

All simple modules in blocks of defect 2 have sources that are not endo-permutation modules. Vertices lie in $\mathrm{Syl}_3(L2_8)$. All other modules lie in blocks of defect 0.

5.2.3 Characteristic 7

Table 5.6: The Group $L_2(8)$ for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_7	V_1	P_1	0	$< 1 >$	Yes
S_7	V_1	P_1	0	$< 1 >$	Yes
S_7	V_1	P_1	0	$< 1 >$	Yes
S_7	V_1	P_1	0	$< 1 >$	Yes
S_8	V_7	P_1	1	C_7	Yes

Here we see that all simple modules have trivial module as source and hence sources are endo-permutation modules.

5.3 The Linear Group $L_2(11)$

The linear group $L_2(7) \cong L_3(2)$ is of order $550 = 2^2 \cdot 3^2 \cdot 5 \cdot 11$ It has standard presentation $< a, b | a^2 = b^3 = (ab)^{11}[a, babab]^2 = 1 >$. We study simple modules in fields of characteristic $p \in \{2, 3, 5, 11\}$ and present our results below.

5.3.1 Characteristic 2

Table 5.7: The Group $L_2(11)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_4	P_1	2	$C_2 \times C_2$	Yes
S_5	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_5	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{10}	V_2	P_1	1	C_2	Yes*
S_{12}	V_1	P_1	0	$< 1 >$	Yes
S_{12}	V_1	P_1	0	$< 1 >$	Yes

Here again we see that all simple modules have sources that are endo-permutation.

5.3.2 Characteristic 3

Table 5.8: The Group $L_2(11)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{5_1}	V_3	P_2	1	C_3	No
S_{5_2}	V_3	P_2	1	C_3	No
S_{10}	V_3	P_1	1	C_3	Yes*
S_{12_1}	V_1	P_1	0	$< 1 >$	Yes
S_{12_2}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_{10}, all modules in block of defect 1 have sources that are not endo-permutation modules. Vertices of these modules lies in $\mathrm{Syl}_3 L_2(11)$.

5.3.3 Characteristic 5

Table 5.9: The Group $L_2(11)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{5_1}	V_1	P_1	0	$< 1 >$	Yes
S_{5_2}	V_1	P_1	0	$< 1 >$	Yes
S_{10_2}	V_1	P_1	0	$< 1 >$	Yes
S_{10_2}	V_1	P_1	0	$< 1 >$	Yes
S_{11}	V_5	P_1	1	C_5	Yes

All modules in this case have sources that are endo-permutation modules.

5.3.4 Characteristic 11

Table 5.10: The Group $L_2(11)$ for characteristic 11

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{11}	P_1	1	C_{11}	Yes
S_3	V_{11}	P_3	1	C_{11}	No
S_5	V_{11}	P_5	1	C_{11}	No
S_7	V_{11}	P_7	1	C_{11}	No
S_9	V_{11}	P_9	1	C_{11}	No
S_{11}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.4 The Linear Group $L_2(13)$

The linear group $L_2(7) \cong L_3(2)$ is of order 1092. We study simple modules in fields of characteristic $p \in \{2, 3, 7, 13\}$ and present our results below.

5.4.1 Characteristic 2

Table 5.11: The Group $L_2(13)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_4	P_1	2	$C_2 \times C_2$	Yes
S_6	V_4	P_2	2	$C_2 \times C_2$	No
S_6	V_4	P_2	2	$C_2 \times C_2$	No
S_{12}	V_1	P_1	0	$< 1 >$	Yes
S_{12}	V_1	P_1	0	$< 1 >$	Yes
S_{12}	V_1	P_1	0	$< 1 >$	Yes
S_{14}	V_2	P_1	1	C_2	Yes*

Only the modules that lie in the principal block of defect 2 have sources that are not end-permutation modules.

5.4.2 Characteristic 3

Table 5.12: The Group $L_2(13)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_7	V_3	P_1	1	C_3	Yes*
S_7	V_3	P_1	1	C_3	Yes*
S_{12}	V_1	P_1	0	$< 1 >$	Yes
S_{12}	V_1	P_1	0	$< 1 >$	Yes
S_{12}	V_1	P_1	0	$< 1 >$	Yes
S_{13}	V_3	P_1	1	C_3	Yes*

This case is another example of all sourcess being endo-permutation.

5.4.3 Characteristic 7

Table 5.13: The Group $L_2(13)$ for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_{7_1}	V_1	P_7	0	$< 1 >$	Yes
S_{7_2}	V_1	P_7	0	$< 1 >$	Yes
S_{12}	V_7	P_5	1	C_7	No
S_{14_1}	V_1	P_1	0	$< 1 >$	Yes
S_{14_2}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_{12}; that lies in block of defect 1; all other modules have sources that are endo-permutation modules.

5.4.4 Characteristic 13

Table 5.14: The Group $L_2(13)$ for characteristic 13

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{13}	P_1	1	C_{13}	Yes
S_3	V_{13}	P_3	1	C_{13}	No
S_5	V_{13}	P_5	1	C_{13}	No
S_7	V_{13}	P_7	1	C_{13}	No
S_9	V_{13}	P_9	1	C_{13}	No
S_{11}	V_{13}	P_11	1	C_{13}	No
S_{13}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.5 The Linear Group $L_2(17)$

The linear group $L_2(17)$ is of order $32448 = 2^4 \cdot 3^2 \cdot 17$. It has presentation $< a, b|a^2 = b^3 = (ab)^{17} = ((ab)^5(ab^{-1})^3)^2 = 1 >$. We study simple modules in fields of characteristic $p \in \{2, 17\}$ and present our results below.

5.5.1 Characteristic 2

Table 5.15: The Group $L_2(17)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{16}	P_1	4	D_{16}	Yes
S_8	V_4	P_4	2	$C_2 \times C_2$	No
S_8	V_4	P_4	2	$C_2 \times C_2$	No
S_{16_1}	V_1	P_1	0	$< 1 >$	Yes
S_{16_2}	V_1	P_1	0	$< 1 >$	Yes
S_{16_3}	V_1	P_1	0	$< 1 >$	Yes
S_{16_4}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 2, have sources that are not endo-permutation modules.All other modules have sources that are endo-permutation.

5.5.2 Characteristic 17

Table 5.16: The Group $L_2(17)$ for characteristic 17

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{17}	P_1	1	C_{17}	Yes
S_3	V_{17}	P_3	1	C_{17}	No
S_5	V_{17}	P_5	1	C_{17}	No
S_7	V_{17}	P_7	1	C_{17}	No
S_9	V_{17}	P_9	1	C_{17}	No
S_{11}	V_{17}	P_11	1	C_{17}	No
S_{13}	V_{17}	P_13	1	C_{17}	No
S_{15}	V_{17}	P_15	1	C_{17}	No
S_{17}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.6 The Linear Group $L_2(19)$

The linear group $L_2(19)$ is of order $3420 = 2^2 \cdot 3^2 \cdot 5 \cdot 19$. We study simple modules in fields of characteristic $p \in \{2, 3, 5, 19\}$ and present our results below.

5.6.1 Characteristic 2

Table 5.17: The Group $L_2(19)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_4	P_1	2	$C_2 \times C_2$	Yes
S_{9_1}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{9_2}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{18_1}	V_2	P_1	1	C_2	Yes*
S_{18_2}	V_2	P_1	1	C_2	Yes*
S_{20_1}	V_1	P_{20}	0	$< 1 >$	Yes
S_{20_3}	V_1	P_1	0	$< 1 >$	Yes
S_{20_4}	V_1	P_1	0	$< 1 >$	Yes
S_{20_5}	V_1	P_1	0	$< 1 >$	Yes

All the modules in this case have sources that are endo-permutation.

5.6.2 Characteristic 3

Table 5.18: The Group $L_2(19)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_9	P_1	2	C_9	Yes
S_{9_1}	V_1	P_1	0	$< 1 >$	Yes
S_{9_2}	V_1	P_1	0	$< 1 >$	Yes
S_{18_1}	V_1	P_1	0	$< 1 >$	Yes
S_{18_2}	V_1	P_1	0	$< 1 >$	Yes
S_{18_3}	V_1	P_1	0	$< 1 >$	Yes
S_{18_4}	V_1	P_1	0	$< 1 >$	Yes
S_{19}	V_9	P_1	2	C_9	Yes

All modules in this case have sources that are endo-permutation modules.

5.6.3 Characteristic 5

Table 5.19: The Group $L_2(19)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{9_1}	V_5	P_4	1	C_5	No
S_{9_2}	V_5	P_4	1	C_5	No
S_{18}	V_5	P_3	1	C_5	No
S_{20_1}	V_1	P_1	0	$< 1 >$	Yes
S_{20_2}	V_1	P_1	0	$< 1 >$	Yes
S_{20_3}	V_1	P_1	0	$< 1 >$	Yes
S_{20_4}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.6.4 Characteristic 19

Table 5.20: The Group $L_2(19)$ for characteristic 19

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{19}	P_1	1	C_{19}	Yes
S_3	V_{19}	P_3	1	C_{19}	No
S_5	V_{19}	P_5	1	C_{19}	No
S_7	V_{19}	P_7	1	C_{19}	No
S_9	V_{19}	P_9	1	C_{19}	No
S_{11}	V_{19}	$P_1 1$	1	C_{19}	No
S_{13}	V_{19}	$P_1 3$	1	C_{19}	No
S_{15}	V_{19}	$P_1 5$	1	C_{19}	No
S_{17}	V_{19}	$P_1 7$	1	C_{19}	No
S_{19}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.7 The Linear Group $L_2(16)$

The linear group $L_2(16)$ is of order $4080 = 2^4 \cdot 3 \cdot 5 \cdot 17$. It has presentation $< a, b | a^2 = b^3 = (ab)^{15} = ((ab)^5(ab^{-1})^3)^2 = 1 >$.We study simple modules in fields of characteristic $p \in \{2, 3, 5, 17\}$ and present our results below.

5.7.1 Characteristic 2

Table 5.21: The Group $L_2(16)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{16}	P_1	4	$C_2 \times C_2 \times C_2 \times C_2$	Yes
S_2	V_{16}	P_2	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_2	V_{16}	P_2	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_2	V_{16}	P_2	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_2	V_{16}	P_2	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_4	V_{16}	P_4	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_4	V_{16}	P_4	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_4	V_{16}	P_4	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_4	V_{16}	P_4	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_4	V_{16}	P_4	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_4	V_{16}	P_4	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_8	V_{16}	P_8	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_8	V_{16}	P_8	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_8	V_{16}	P_8	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_8	V_{16}	P_8	4	$C_2 \times C_2 \times C_2 \times C_2$	No
S_{16}	V_1	P_{16}	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 4, have sources that are not endo-permutation modules.

5.7.2 Characteristic 3

Table 5.22: The Group $L_2(16)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{15_1}	V_1	P_1	0	$< 1 >$	Yes
S_{15_2}	V_1	P_1	0	$< 1 >$	Yes
S_{15_3}	V_1	P_1	0	$< 1 >$	Yes
S_{15_4}	V_1	P_1	0	$< 1 >$	Yes
S_{15_5}	V_1	P_1	0	$< 1 >$	Yes
S_{15_6}	V_1	P_1	0	$< 1 >$	Yes
S_{15_7}	V_1	P_1	0	$< 1 >$	Yes
S_{15_8}	V_1	P_1	0	$< 1 >$	Yes
S_{16}	V_3	P_1	1	C_3	Yes*
S_{17}	V_3	P_1	1	C_3	Yes*
S_{17}	V_3	P_1	1	C_3	Yes*

All modules in this case sources that are endo-permutation modules.

5.7.3 Characteristic 5

Table 5.23: The Group $L_2(16)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{15_1}	V_1	P_1	0	$< 1 >$	Yes
S_{15_2}	V_1	P_1	0	$< 1 >$	Yes
S_{15_3}	V_1	P_1	0	$< 1 >$	Yes
S_{15_4}	V_1	P_1	0	$< 1 >$	Yes
S_{15_5}	V_1	P_1	0	$< 1 >$	Yes
S_{15_6}	V_1	P_1	0	$< 1 >$	Yes
S_{15_7}	V_1	P_1	0	$< 1 >$	Yes
S_{15_8}	V_1	P_1	0	$< 1 >$	Yes
S_{16}	V_5	P_1	1	C_5	Yes*
S_{17}	V_5	P_1	1	C_5	Yes*

All the modules in this case have trivial module as their source and are hence endo-permutation.

5.7.4 Characteristic 17

Table 5.24: The Group $L_2(16)$ for characteristic 17

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{17}	P_1	1	C_{17}	Yes
S_{15}	V_{17}	P_15	1	C_{17}	No
S_{17}	V_1	P_1	0	$< 1 >$	Yes
S_{17}	V_1	P_1	0	$< 1 >$	Yes
S_{17}	V_1	P_1	0	$< 1 >$	Yes
S_{17}	V_1	P_1	0	$< 1 >$	Yes
S_{17}	V_1	P_1	0	$< 1 >$	Yes
S_{17}	V_1	P_1	0	$< 1 >$	Yes
S_{17}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.8 The Linear Group $L_2(23)$

The linear group $L_2(23)$ is of order 6072. We study simple modules in fields of characteristic $p \in \{2, 3, 11, 13\}$ and present our results below.

5.8.1 Characteristic 2

Table 5.25: The Group $L_2(23)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_8	P_1	3	D_8	Yes
S_{11_1}	V_8	P_3	3	D_8	No
S_{11_2}	V_8	P_3	3	D_8	No
S_{22}	V_4	P_3	2	$C_2 \times C_2$	No
S_{24_1}	V_1	P_1	0	$< 1 >$	Yes
S_{24_2}	V_1	P_1	0	$< 1 >$	Yes
S_{24_3}	V_1	P_1	0	$< 1 >$	Yes
S_{24_4}	V_1	P_1	0	$< 1 >$	Yes
S_{24_5}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 3, and modules in blocks with defect 2 have sources that are not endo-permutation modules.

5.8.2 Characteristic 3

Table 5.26: The Group $L_2(23)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{11_1}	V_3	P_2	1	C_3	No
S_{11_2}	V_3	P_2	1	C_3	No
S_{22_1}	V_3	P_1	1	C_3	Yea*
S_{22_2}	V_3	P_2	1	C_3	No
S_{24_1}	V_1	P_1	0	$< 1 >$	Yes
S_{24_2}	V_1	P_1	0	$< 1 >$	Yes
S_{24_3}	V_1	P_1	0	$< 1 >$	Yes
S_{24_4}	V_1	P_1	0	$< 1 >$	Yes
S_{24_5}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_{22_1}, all modules in the principal block have sources that are not endo-permutation modules.

5.8.3 Characteristic 11

Table 5.27: The Group $L_2(23)$ for characteristic 11

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{11}	P_1	1	C_{11}	Yes
S_{11_1}	V_1	P_1	0	$< 1 >$	Yes
S_{11_2}	V_1	P_1	0	$< 1 >$	Yes
S_{22_1}	V_1	P_1	0	$< 1 >$	Yes
S_{22_2}	V_1	P_1	0	$< 1 >$	Yes
S_{22_3}	V_1	P_1	0	$< 1 >$	Yes
S_{22_4}	V_1	P_1	0	$< 1 >$	Yes
S_{22_5}	V_1	P_1	0	$< 1 >$	Yes
S_{23}	V_{11}	P_1	1	C_{11}	Yes

All modules that lie in this case have sources that are endo-permutation modules.

5.8.4 Characteristic 23

Table 5.28: The Group $L_2(23)$ for characteristic 23

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{23}	P_1	1	C_{23}	Yes
S_3	V_{23}	P_3	1	C_{23}	No
S_5	V_{23}	P_5	1	C_{23}	No
S_7	V_{23}	P_7	1	C_{23}	No
S_9	V_{23}	P_9	1	C_{23}	No
S_{11}	V_{23}	P_11	1	C_{23}	No
S_{13}	V_{23}	P_13	1	C_{23}	No
S_{15}	V_{23}	P_15	1	C_{23}	No
S_{17}	V_{23}	P_17	1	C_{23}	No
S_{19}	V_{23}	P_19	1	C_{23}	No
S_{21}	V_{23}	P_21	1	C_{23}	No
S_{23}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.9 The Linear Group $L_2(25)$

5.9.1 Characteristic 2

Table 5.29: The Group $L_2(25)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_8	P_1	3	D_8	Yes
S_{12}	V_4	P_4	2	$C_2 \times C_2$	No
S_{12}	V_4	P_4	2	$C_2 \times C_2$	No
S_{24_1}	V_1	P_1	0	$< 1 >$	Yes
S_{24_2}	V_1	P_1	0	$< 1 >$	Yes
S_{24_3}	V_1	P_1	0	$< 1 >$	Yes
S_{24_4}	V_1	P_1	0	$< 1 >$	Yes
S_{24_5}	V_1	P_1	0	$< 1 >$	Yes
S_{24_6}	V_1	P_1	0	$< 1 >$	Yes
S_{26_6}	V_4	P_1	2	$C_2 \times C_2$	Yes*

All modules that lie in the block with defect 2 except S_{26_6} have sources that are not endo-permutation modules.

5.9.2 Characteristic 3

Table 5.30: The Group $L_2(25)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{13_1}	V_1	P_{13}	0	$< 1 >$	Yes
S_{13_2}	V_3	P_1	1	C_3	Yes*
S_{24_1}	V_1	P_1	0	$< 1 >$	Yes
S_{24_2}	V_1	P_1	0	$< 1 >$	Yes
S_{24_3}	V_1	P_1	0	$< 1 >$	Yes
S_{24_4}	V_1	P_1	0	$< 1 >$	Yes
S_{24_5}	V_1	P_1	0	$< 1 >$	Yes
S_{24_5}	V_1	P_1	0	$< 1 >$	Yes
S_{25}	V_3	P_1	1	C_3	Yes*
S_{26}	V_3	P_1	1	C_3	Yes*

All modules have sources that are endo-permutation modules.

5.9.3 Characteristic 5

Table 5.31: The Group $L_2(25)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{25}	P_1	2	$C_5 \times C_5$	Yes
S_{3_1}	V_{25}	P_3	2	$C_5 \times C_5$	No
S_{3_2}	V_{25}	P_3	2	$C_5 \times C_5$	No
S_4	V_{25}	P_4	2	$C_5 \times C_5$	No
S_{5_1}	V_{25}	P_5	2	$C_5 \times C_5$	No
S_{5_2}	V_{25}	P_5	2	$C_5 \times C_5$	No
S_{8_1}	V_{25}	P_8	2	$C_5 \times C_5$	No
S_{8_2}	V_{25}	P_8	2	$C_5 \times C_5$	No
S_9	V_{25}	P_9	2	$C_5 \times C_5$	No
S_{15_1}	V_{25}	P_15	2	$C_5 \times C_5$	No
S_{15_2}	V_{25}	P_15	2	$C_5 \times C_5$	No
S_{16}	V_{25}	P_16	2	$C_5 \times C_5$	No
S_{25}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 2, have sources that are not endo-permutation modules.

5.9.4 Characteristic 13

Table 5.32: The Group $L_2(25)$ for characteristic 13

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{13}	P_1	1	C_13	Yes
S_{13}	V_1	P_1	0	$< 1 >$	Yes
S_{13}	V_1	P_1	0	$< 1 >$	Yes
S_{24}	V_{13}	P_{11}	1	C_{13}	No
S_{26_1}	V_1	P_1	0	$< 1 >$	Yes
S_{26_2}	V_1	P_1	0	$< 1 >$	Yes
S_{26_3}	V_1	P_1	0	$< 1 >$	Yes
S_{26_4}	V_1	P_1	0	$< 1 >$	Yes
S_{26_5}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.10 The Linear Group $L_2(27)$

The linear group $L_2(27)$ is of order $9828 = 2^2 \cdot 3^3 \cdot 7 \cdot 13$. We study simple modules in fields of characteristic $p \in \{2, 3, 7, 13\}$ and present our results below.

5.10.1 Characteristic 2

Table 5.33: The Group $L_2(27)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_4	P_1	2	$C_2 \times C_2$	Yes
S_{13_1}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{13_2}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{26_1}	V_2	P_1	1	C_2	Yes*
S_{26_2}	V_2	P_1	1	C_2	Yes*
S_{26_3}	V_2	P_2	1	C_2	No
S_{28_1}	V_1	P_1	0	$< 1 >$	Yes
S_{28_2}	V_1	P_1	0	$< 1 >$	Yes
S_{28_3}	V_1	P_1	0	$< 1 >$	Yes
S_{28_4}	V_1	P_1	0	$< 1 >$	Yes
S_{28_5}	V_1	P_1	0	$< 1 >$	Yes
S_{28_6}	V_1	P_1	0	$< 1 >$	Yes

With the exception of S_{26_3} all the modules have sources that are endo-permutation.

5.10.2 Characteristic 3

Table 5.34: The Group $L_2(27)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{27}	P_1	3	$C_3 \times C_3 \times C_3$	Yes
S_{3_1}	V_{27}	P_3	3	$C_3 \times C_3 \times C_3$	No
S_{3_2}	V_{27}	P_3	3	$C_3 \times C_3 \times C_3$	No
S_{3_3}	V_{27}	P_3	3	$C_3 \times C_3 \times C_3$	No
S_{4_1}	V_{27}	P_4	3	$C_3 \times C_3 \times C_3$	No
S_{4_2}	V_{27}	P_4	3	$C_3 \times C_3 \times C_3$	No
S_{4_3}	V_{27}	P_4	3	$C_3 \times C_3 \times C_3$	No
S_{9_1}	V_{27}	P_9	3	$C_3 \times C_3 \times C_3$	No
S_{9_2}	V_{27}	P_9	3	$C_3 \times C_3 \times C_3$	No
S_{12_1}	V_{27}	P_{12}	3	$C_3 \times C_3 \times C_3$	No
S_{12_2}	V_{27}	P_{12}	3	$C_3 \times C_3 \times C_3$	No
S_{12_3}	V_{27}	P_{12}	3	$C_3 \times C_3 \times C_3$	No
S_{27}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 3, have sources that are not endo-permutation modules.

5.10.3 Characteristic 7

Table 5.35: The Group $L_2(27)$ for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_{13_1}	V_7	P_6	1	C_7	No
S_{13_2}	V_7	P_6	1	C_7	No
S_{26}	V_7	P_5	1	C_7	No
S_{28_1}	V_1	P_1	0	$< 1 >$	Yes
S_{28_2}	V_1	P_1	0	$< 1 >$	Yes
S_{28_3}	V_1	P_1	0	$< 1 >$	Yes
S_{28_4}	V_1	P_1	0	$< 1 >$	Yes
S_{28_5}	V_1	P_1	0	$< 1 >$	Yes
S_{28_6}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.10.4 Characteristic 13

Table 5.36: The Group $L_2(27)$ for characteristic 13

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{13}	P_1	1	C_{13}	Yes
S_{13_1}	V_1	P_1	0	$< 1 >$	Yes
S_{13_2}	V_1	P_1	0	$< 1 >$	Yes
S_{26_1}	V_1	P_1	0	$< 1 >$	Yes
S_{26_2}	V_1	P_1	0	$< 1 >$	Yes
S_{26_3}	V_1	P_1	0	$< 1 >$	Yes
S_{26_4}	V_1	P_1	0	$< 1 >$	Yes
S_{26_5}	V_1	P_1	0	$< 1 >$	Yes
S_{26_6}	V_1	P_1	0	$< 1 >$	Yes
S_{27}	V_{13}	P_1	1	C_{13}	Yes*

All modules have sources that are endo-permutation modules.

5.11 The Linear Group $L_2(29)$

The linear group $L_2(20)$ is of order 12180. We study simple modules in fields of characteristic $p \in \{2, 3, 5, 7\}$ and present our results below.

5.11.1 Characteristic 2

Table 5.37: The Group $L_2(29)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_4	P_1	2	$C_2 \times C_2$	Yes
S_{14_1}	V_4	P_2	2	$C_2 \times C_2$	No
S_{14_2}	V_4	P_2	2	$C_2 \times C_2$	No
S_{28_1}	V_1	P_1	0	$< 1 >$	Yes
S_{28_2}	V_1	P_1	0	$< 1 >$	Yes
S_{28_3}	V_1	P_1	0	$< 1 >$	Yes
S_{28_4}	V_1	P_1	0	$< 1 >$	Yes
S_{28_5}	V_1	P_1	0	$< 1 >$	Yes
S_{28_6}	V_1	P_1	0	$< 1 >$	Yes
S_{28_7}	V_1	P_1	0	$< 1 >$	Yes
S_{30_1}	V_2	P_1	1	C_2	Yes*
S_{30_2}	V_2	P_2	1	C_2	No
S_{30_3}	V_2	P_2	1	C_2	No

All modules that lie in the principal block (with defect 2) and blocks with defect 1 except S_{30_1} have sources that are not endo-permutation modules.

5.11.2 Characteristic 3

Table 5.38: The Group $L_2(29)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{15_1}	V_1	P_1	0	$< 1 >$	Yes
S_{15_2}	V_1	P_1	0	$< 1 >$	Yes
S_{28_1}	V_3	P_1	1	C_3	Yes
S_{28_2}	V_3	P_2	1	C_3	No
S_{28_3}	V_3	P_2	1	C_3	No
S_{30_1}	V_1	P_1	0	$< 1 >$	Yes
S_{30_2}	V_1	P_1	0	$< 1 >$	Yes
S_{30_3}	V_1	P_1	0	$< 1 >$	Yes
S_{30_4}	V_1	P_1	0	$< 1 >$	Yes
S_{30_5}	V_1	P_1	0	$< 1 >$	Yes
S_{30_6}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.11.3 Characteristic 5

Table 5.39: The Group $L_2(29)$ for characteristic 5

Module	Vertex	Source	Defect Group	Endo-permutation?	
S_1	V_5	P_1	1	C_5	Yes
S_{15}	V_1	P_{15}	0	$< 1 >$	Yes
S_{15}	V_1	P_{15}	0	$< 1 >$	Yes
S_{28}	V_5	P_1	1	C_5	Yes
S_{28}	V_5	P_2	1	C_5	Yes
S_{30}	V_1	P_{30}	0	$< 1 >$	Yes
S_{30}	V_1	P_{30}	0	$< 1 >$	Yes
S_{30}	V_1	P_{30}	0	$< 1 >$	Yes
S_{30}	V_1	P_{30}	0	$< 1 >$	Yes
S_{30}	V_1	P_{30}	0	$< 1 >$	Yes
S_{30}	V_1	P_{30}	0	$< 1 >$	Yes

All modules that have sources that not endo-permutation modules.

5.11.4 Characteristic 7

Table 5.40: The Group $L_2(29)$ for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_{15_1}	V_7	P_1	1	C_7	Yes*
S_{15_2}	V_7	P_1	1	C_7	Yes*
S_{28_1}	V_1	P_1	0	$< 1 >$	Yes
S_{28_2}	V_1	P_1	0	$< 1 >$	Yes
S_{28_3}	V_1	P_1	0	$< 1 >$	Yes
S_{28_4}	V_1	P_1	0	$< 1 >$	Yes
S_{28_5}	V_1	P_1	0	$< 1 >$	Yes
S_{28_6}	V_1	P_1	0	$< 1 >$	Yes
S_{28_7}	V_1	P_1	0	$< 1 >$	Yes
S_{29}	V_7	P_1	1	C_7	Yes*

All modules have sources that are endo-permutation modules.

5.12 The Linear Group $L_2(31)$

The linear group $L_2(31)$ is of order $14880 = 2^5 \cdot 3 \cdot 5 \cdot 31$.It has presentation $< a, b | a^2 = b^3 = [a, b]^3 ab[a, b][a, bab][a, b^{-1}abab] = 1 >$. We study simple modules in fields of characteristic $p \in \{2, 3, 5, 31\}$ and present our results below.

5.12.1 Characteristic 2

Table 5.41: The Group $L_2(31)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{32}	P_1	5	D_{32}	Yes
S_{15_1}	V_{32}	P_{15}	5	D_{32}	No
S_{15_2}	V_{32}	P_{15}	5	D_{32}	No
S_{32_1}	V_1	P_1	0	$< 1 >$	Yes
S_{32_2}	V_1	P_1	0	$< 1 >$	Yes
S_{32_3}	V_1	P_1	0	$< 1 >$	Yes
S_{32_4}	V_1	P_1	0	$< 1 >$	Yes
S_{32_5}	V_1	P_1	0	$< 1 >$	Yes
S_{32_5}	V_1	P_1	0	$< 1 >$	Yes
S_{32_6}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 5, have sources that are not endo-permutation modules.

5.12.2 Characteristic 3

Table 5.42: The Group $L_2(31)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{15_1}	V_1	P_1	0	$<1>$	Yes
S_{15_2}	V_1	P_1	0	$<1>$	Yes
S_{30_1}	V_1	P_1	0	$<1>$	Yes
S_{30_2}	V_1	P_1	0	$<1>$	Yes
S_{30_3}	V_1	P_1	0	$<1>$	Yes
S_{30_4}	V_1	P_1	0	$<1>$	Yes
S_{30_5}	V_1	P_1	0	$<1>$	Yes
S_{30_6}	V_1	P_1	0	$<1>$	Yes
S_{30_7}	V_1	P_1	0	$<1>$	Yes
S_{31}	V_3	P_1	1	C_3	Yes*
S_{32_1}	V_3	P_1	1	C_3	Yes*
S_{32_2}	V_3	P_1	1	C_3	Yes*

All modules have sources that are endo-permutation modules.

5.12.3 Characteristic 5

Table 5.43: The Group $L_2(31)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{15_1}	V_1	P_1	0	$<1>$	Yes
S_{15_2}	V_1	P_1	0	$<1>$	Yes
S_{30_1}	V_1	P_1	0	$<1>$	Yes
S_{30_2}	V_1	P_1	0	$<1>$	Yes
S_{30_3}	V_1	P_1	0	$<1>$	Yes
S_{30_4}	V_1	P_1	0	$<1>$	Yes
S_{30_5}	V_1	P_1	0	$<1>$	Yes
S_{30_6}	V_1	P_1	0	$<1>$	Yes
S_{30_7}	V_1	P_1	0	$<1>$	Yes
S_{31}	V_5	P_1	1	C_5	Yes*
S_{32}	V_5	P_1	1	C_5	Yes*

All modules have sources that are endo-permutation modules.

5.12.4 Characteristic 31

Table 5.44: The Group $L_2(31)$ for characteristic 31

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{31}	P_1	1	C_{31}	Yes
S_3	V_{31}	P_3	1	C_{31}	No
S_5	V_{31}	P_5	1	C_{31}	No
S_7	V_{31}	P_7	1	C_{31}	No
S_9	V_{31}	P_9	1	C_{31}	No
S_{11}	V_{31}	P_{11}	1	C_{31}	No
S_{13}	V_{31}	P_{13}	1	C_{31}	No
S_{15}	V_{31}	P_{15}	1	C_{31}	No
S_{17}	V_{31}	P_{17}	1	C_{31}	No
S_{19}	V_{31}	P_{19}	1	C_{31}	No
S_{21}	V_{31}	P_{21}	1	C_{31}	No
S_{23}	V_{31}	P_{23}	1	C_{31}	No
S_{25}	V_{31}	P_{25}	1	C_{31}	No
S_{27}	V_{31}	P_{27}	1	C_{31}	No
S_{29}	V_{31}	P_{29}	1	C_{31}	No
S_{31}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.13 The Linear Group $L_2(32)$

The linear group $L_2(32)$ is of order $32736 = 2^5 \cdot 3 \cdot 11 \cdot 31$.It has presentation $< a, b | a^2 = b^3 = (ab)^{31} = (ab)^4(ab^{-1}abab^{-1})^3(ab)^4(ab^{-1})^2 = 1 >$. We study simple modules in fields of characteristic $p \in \{2, 3, 11, 31\}$ and present our results below.

5.13.1 Characteristic 2

Table 5.45: The Group $L_2(32)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{32}	P_1	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	Yes
S_{2_1}	V_{32}	P_2	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{2_2}	V_{32}	P_2	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{2_3}	V_{32}	P_2	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{2_4}	V_{32}	P_2	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{2_5}	V_{32}	P_2	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{4_1}	V_{32}	P_4	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{4_2}	V_{32}	P_4	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{4_3}	V_{32}	P_4	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{4_4}	V_{32}	P_4	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{4_5}	V_{32}	P_4	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{4_6}	V_{32}	P_4	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{4_7}	V_{32}	P_4	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{4_8}	V_{32}	P_4	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{4_9}	V_{32}	P_4	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
$S_{4_{10}}$	V_{32}	P_4	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{8_1}	V_{32}	P_8	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{8_2}	V_{32}	P_8	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{8_3}	V_{32}	P_8	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{8_4}	V_{32}	P_8	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{8_5}	V_{32}	P_8	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{8_6}	V_{32}	P_8	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{8_7}	V_{32}	P_8	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{8_8}	V_{32}	P_8	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{8_9}	V_{32}	P_8	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
$S_{8_{10}}$	V_{32}	P_8	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{16_1}	V_{32}	P_{16}	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{16_2}	V_{32}	P_{16}	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{16_3}	V_{32}	P_{16}	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{16_4}	V_{32}	P_{16}	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{16_5}	V_{32}	P_{16}	5	$C_2 \times C_2 \times C_2 \times C_2 \times C_2$	No
S_{32}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 5, have sources that are not endo-permutation modules.

5.13.2 Characteristic 3

Table 5.46: The Group $L_2(32)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{31_1}	V_3	P_1	1	C_3	Yes*
S_{31_2}	V_3	P_2	1	C_3	No
S_{31_3}	V_3	P_2	1	C_3	No
S_{31_4}	V_3	P_2	1	C_3	No
S_{31_5}	V_3	P_2	1	C_3	No
S_{31_6}	V_3	P_2	1	C_3	No
S_{33_1}	V_1	P_1	0	$< 1 >$	Yes
S_{33_2}	V_1	P_1	0	$< 1 >$	Yes
S_{33_3}	V_1	P_1	0	$< 1 >$	Yes
S_{33_4}	V_1	P_1	0	$< 1 >$	Yes
S_{33_5}	V_1	P_1	0	$< 1 >$	Yes
S_{33_6}	V_1	P_1	0	$< 1 >$	Yes
S_{33_7}	V_1	P_1	0	$< 1 >$	Yes
S_{33_8}	V_1	P_1	0	$< 1 >$	Yes
S_{33_9}	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{10}}$	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{11}}$	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{12}}$	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{13}}$	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{14}}$	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{15}}$	V_1	P_1	0	$< 1 >$	Yes

All modules except S_{31_1} that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.13.3 Characteristic 11

Table 5.47: The Group $L_2(32)$ for characteristic 11

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{11}	P_1	1	C_{11}	Yes
S_{31_1}	V_{11}	P_9	1	C_{11}	No
S_{31_2}	V_{11}	P_9	1	C_{11}	No
S_{33_1}	V_1	P_1	0	$< 1 >$	Yes
S_{33_2}	V_1	P_1	0	$< 1 >$	Yes
S_{33_3}	V_1	P_1	0	$< 1 >$	Yes
S_{33_4}	V_1	P_1	0	$< 1 >$	Yes
S_{33_5}	V_1	P_1	0	$< 1 >$	Yes
S_{33_6}	V_1	P_1	0	$< 1 >$	Yes
S_{33_7}	V_1	P_1	0	$< 1 >$	Yes
S_{33_8}	V_1	P_1	0	$< 1 >$	Yes
S_{33_9}	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{10}}$	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{11}}$	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{12}}$	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{13}}$	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{14}}$	V_1	P_1	0	$< 1 >$	Yes
$S_{33_{15}}$	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

5.13.4 Characteristic 31

Table 5.48: The Group $L_2(32)$ for characteristic 31

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{31}	P_1	1	C_{31}	Yes
S_{31_1}	V_1	P_{31}	0	$< 1 >$	Yes
S_{31_2}	V_1	P_{31}	0	$< 1 >$	Yes
S_{31_3}	V_1	P_{31}	0	$< 1 >$	Yes
S_{31_4}	V_1	P_{31}	0	$< 1 >$	Yes
S_{31_5}	V_1	P_{31}	0	$< 1 >$	Yes
S_{31_6}	V_1	P_{31}	0	$< 1 >$	Yes
S_{31_7}	V_1	P_{31}	0	$< 1 >$	Yes
S_{31_8}	V_1	P_{31}	0	$< 1 >$	Yes
S_{31_9}	V_1	P_{31}	0	$< 1 >$	Yes
$S_{31_{10}}$	V_1	P_{31}	0	$< 1 >$	Yes
$S_{31_{11}}$	V_1	P_{31}	0	$< 1 >$	Yes
$S_{31_{12}}$	V_1	P_{31}	0	$< 1 >$	Yes
$S_{31_{13}}$	V_1	P_{31}	0	$< 1 >$	Yes
$S_{31_{14}}$	V_1	P_{31}	0	$< 1 >$	Yes
$S_{31_{15}}$	V_1	P_{31}	0	$< 1 >$	Yes
$S_{31_{16}}$	V_1	P_{31}	0	$< 1 >$	Yes
S_{32}	V_{31}	P_1	1	C_{31}	Yes*

All modules have sources that are endo-permutation modules.

5.14 Linear Group $L_3(3)$

Linear group $L_3(3)$ is of order $5616 = 2^4 \cdot 3^3 \cdot 13$. We study simple modules in fields of characteristic $p \in \{2\}$ and present our results below.

5.14.1 Characteristic 2

Table 5.49: The Group $L_3(3)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{16}	P_1	4	QD_{16}	Yes
S_{12}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{16_1}	V_1	P_1	0	$< 1 >$	Yes
S_{16_2}	V_1	P_1	0	$< 1 >$	Yes
S_{16_3}	V_1	P_1	0	$< 1 >$	Yes
S_{16_4}	V_1	P_1	0	$< 1 >$	Yes
S_{26}	V_8	P_1	3	Q_8	Yes*

All modules have sources that are not endo-permutation modules.

Chapter 6

Some Other Simple Groups

In this chapter, we continue or study for some more simple groups.

6.1 Linear Groups continued

6.1.1 The Linear Group $L_3(3)$

The linear group $L_3(3)$ is of order $5616 = 2^4 \cdot 3^3 \cdot 13$ We study simple modules in fields of characteristic $p \in \{3, 13\}$ and present our results below.

Characteristic 3

Table 6.1: The Group $L_3(3)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{27}	P_1	3	$(C_3 \times C_3) : C_3$	Yes
S_3	V_{27}	P_3	3	$(C_3 \times C_3) : C_3$	No
S_3	V_{27}	P_3	3	$(C_3 \times C_3) : C_3$	No
S_6	V_{27}	P_6	3	$(C_3 \times C_3) : C_3$	No
S_6	V_{27}	P_6	3	$(C_3 \times C_3) : C_3$	No
S_7	V_{27}	P_7	3	$(C_3 \times C_3) : C_3$	No
S_{15}	V_{27}	P_{15}	3	$(C_3 \times C_3) : C_3$	No
S_{15}	V_{27}	P_{15}	3	$(C_3 \times C_3) : C_3$	No
S_{27}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 3, have sources that are not endo-permutation modules.

Characteristic 13

Table 6.2: The Group $L_3(3)$ for characteristic 13

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{13}	P_1	1	C_{13}	Yes
S_{11}	V_{13}	P_{11}	1	C_{13}	Yes*
S_{13}	V_1	P_1	0	$< 1 >$	Yes
S_{16}	V_{13}	P_3	1	C_{13}	No
S_{26_1}	V_1	P_1	0	$< 1 >$	Yes
S_{26_2}	V_1	P_1	0	$< 1 >$	Yes
S_{26_3}	V_1	P_1	0	$< 1 >$	Yes
S_{39}	V_1	P_1	0	$< 1 >$	Yes

All modules except S_{11} that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

6.1.2 The Linear Group $L_3(4)$

The linear group $L_3(4)$ is of order $20160 = 2^6 \cdot 3^2 \cdot 5 \cdot 7$ It has standard presentation $< a, b | a^2 = b^4 = (ab)^7 = (ab^2)^5 = (abab^2)^7 = (ababab^2ab^{-1})^5 = 1 >$. We study simple modules in fields of characteristic $p \in \{2, 3, 5, 7\}$ and present our results below.

Characteristic 2

Table 6.3: The Group $L_3(4)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	EP?
S_1	V_{64}	P_1	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	Yes
S_8	V_{64}	P_8	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_8	V_{64}	P_8	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_9	V_{64}	P_9	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_9	V_{64}	P_9	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{64}	V_1	P_{64}	0	$< 1 >$	Yes

Read EP as Endo-permutation All modules that lie in the principal block, with defect 6, have sources that are not endo-permutation modules.

Characteristic 3

Table 6.4: The Group $L_3(4)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{15_1}	V_9	P_6	2	$C_3 \times C_3$	No
S_{15_2}	V_9	P_6	2	$C_3 \times C_3$	No
S_{15_3}	V_9	P_6	2	$C_3 \times C_3$	No
S_{19}	V_9	P_{10}	2	$C_3 \times C_3$	No
S_{45_1}	V_1	P_1	0	$< 1 >$	Yes
S_{45_2}	V_1	P_1	0	$< 1 >$	Yes
S_{63_1}	V_1	P_1	0	$< 1 >$	Yes
S_{63_2}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 2, have sources that are not endo-permutation modules.

Characteristic 5

Table 6.5: The Group $L_3(4)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{20}	V_1	P_1	0	$< 1 >$	Yes
S_{35_1}	V_1	P_1	0	$< 1 >$	Yes
S_{35_2}	V_1	P_1	0	$< 1 >$	Yes
S_{35_3}	V_1	P_1	0	$< 1 >$	Yes
S_{45_1}	V_1	P_1	0	$< 1 >$	Yes
S_{45_2}	V_1	P_1	0	$< 1 >$	Yes
S_{63}	V_5	P_3	1	C_5	No

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

Characteristic 7

Table 6.6: The Group $L_3(4)$ for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_{19}	V_7	P_5	1	C_7	No
S_{35_1}	V_1	P_1	0	$< 1 >$	Yes
S_{35_2}	V_1	P_1	0	$< 1 >$	Yes
S_{35_3}	V_1	P_1	0	$< 1 >$	Yes
S_{45}	V_7	P_3	1	C_7	No
S_{63_1}	V_1	P_1	0	$< 1 >$	Yes
S_{63_2}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

6.1.3 The Linear Group $L_3(5)$

The linear group $L_3(5)$ is of order $372000 = 2^5 \cdot 3 \cdot 5^3 \cdot 31$ It has standard presentation $< a, b | a^3 = b^3 = aba^{-1}baba^{-1}b^2ab^{-2}a^{-1}b^2 = abab^{-2}(a^{-1}b^2a^{-1}b^{-2})^3 = 1 >$. We study simple modules in fields of characteristic $p \in \{2, 3, 5, 31\}$ and present our results below.

Characteristic 2

Table 6.7: The Group $L_3(5)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{32}	P_1	5	$(C_4 \times C_4) : C_2$	Yes
S_{30}	V_{16}	P_1	4	$C_4 \times C_4$	Yes
S_{96_1}	V_1	P_1	0	$< 1 >$	Yes
S_{96_2}	V_1	P_1	0	$< 1 >$	Yes
S_{96_3}	V_1	P_1	0	$< 1 >$	Yes
S_{96_4}	V_1	P_1	0	$< 1 >$	Yes
S_{96_5}	V_1	P_1	0	$< 1 >$	Yes
S_{96_6}	V_1	P_1	0	$< 1 >$	Yes
S_{96_7}	V_1	P_1	0	$< 1 >$	Yes
S_{96_8}	V_1	P_1	0	$< 1 >$	Yes
S_{96_9}	V_1	P_1	0	$< 1 >$	Yes
$S_{96_{10}}$	V_1	P_1	0	$< 1 >$	Yes
S_{124_1}	V_8	P_1	3	C_8	Yes*
S_{124_2}	V_{16}	P_4	4	$C_4 \times C_4$	No

All modules that block with defect 4 have sources that are not endo-permutation modules., while in all other cases, source modules are endo-permutation modules.

Characteristic 3

Table 6.8: The Group $L_3(5)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{30}	V_1	P_{30}	0	$< 1 >$	Yes
S_{31_1}	V_3	P_1	1	C_3	Yes*
S_{31_2}	V_3	P_1	1	C_3	Yes*
S_{31_3}	V_3	P_1	1	C_3	Yes*
S_{96_1}	V_1	P_{96}	0	$< 1 >$	Yes
S_{96_2}	V_1	P_{96}	0	$< 1 >$	Yes
S_{96_3}	V_1	P_{96}	0	$< 1 >$	Yes
S_{96_4}	V_1	P_{96}	0	$< 1 >$	Yes
S_{96_5}	V_1	P_{96}	0	$< 1 >$	Yes
S_{96_6}	V_1	P_{96}	0	$< 1 >$	Yes
S_{96_7}	V_1	P_{96}	0	$< 1 >$	Yes
S_{96_8}	V_1	P_{96}	0	$< 1 >$	Yes
S_{96_9}	V_1	P_{96}	0	$< 1 >$	Yes
$S_{96_{10}}$	V_1	P_{96}	0	$< 1 >$	Yes
S_{124_1}	V_3	P_1	1	C_3	Yes*
S_{124_2}	V_3	P_1	1	C_3	Yes*
S_{124_3}	V_3	P_2	1	C_3	No
S_{124_4}	V_3	P_2	1	C_3	No
S_{124_5}	V_3	P_1	1	C_3	Yes*
S_{124_6}	V_3	P_1	1	C_3	Yes*
S_{186}	V_1	P_{186}	0	$< 1 >$	Yes

All modules have sources that are endo-permutation modules.

Characteristic 5

Table 6.9: The Group $L_3(5)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{125}	P_1	3	$(C_5 \times C_5) : C_5$	Yes
S_{3_1}	V_{125}	P_3	3	$(C_5 \times C_5) : C_5$	No
S_{3_2}	V_{125}	P_3	3	$(C_5 \times C_5) : C_5$	No
S_{6_1}	V_{125}	P_6	3	$(C_5 \times C_5) : C_5$	No
S_{6_2}	V_{125}	P_6	3	$(C_5 \times C_5) : C_5$	No
S_8	V_{125}	P_8	3	$(C_5 \times C_5) : C_5$	No
S_{10_1}	V_{125}	P_{10}	3	$(C_5 \times C_5) : C_5$	No
S_{10_2}	V_{125}	P_{10}	3	$(C_5 \times C_5) : C_5$	No
S_{15_1}	V_{125}	P_{15}	3	$(C_5 \times C_5) : C_5$	No
S_{15_2}	V_{125}	P_{15}	3	$(C_5 \times C_5) : C_5$	No
S_{15_3}	V_{125}	P_{15}	3	$(C_5 \times C_5) : C_5$	No
S_{15_4}	V_{125}	P_{15}	3	$(C_5 \times C_5) : C_5$	No
S_{18_1}	V_{125}	P_{18}	3	$(C_5 \times C_5) : C_5$	No
S_{18_2}	V_{125}	P_{18}	3	$(C_5 \times C_5) : C_5$	No
S_{19}	V_{125}	P_{19}	3	$(C_5 \times C_5) : C_5$	No
S_{35_1}	V_{125}	P_{35}	3	$(C_5 \times C_5) : C_5$	No
S_{35_2}	V_{125}	P_{35}	3	$(C_5 \times C_5) : C_5$	No
S_{39_1}	V_{125}	P_{39}	3	$(C_5 \times C_5) : C_5$	No
S_{39_2}	V_{125}	P_{39}	3	$(C_5 \times C_5) : C_5$	No
S_{60_1}	V_{125}	P_{60}	3	$(C_5 \times C_5) : C_5$	No
S_{60_2}	V_{125}	P_{60}	3	$(C_5 \times C_5) : C_5$	No
S_{63}	V_{125}	P_{63}	3	$(C_5 \times C_5) : C_5$	No
S_{90_1}	V_{125}	P_{90}	3	$(C_5 \times C_5) : C_5$	No
S_{90_2}	V_{125}	P_{90}	3	$(C_5 \times C_5) : C_5$	No
S_{125}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 3, have sources that are not endo-permutation modules.

Characteristic 31

Table 6.10: The Group $L_3(5)$ for characteristic 31

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{31}	P_1	1	C_{31}	Yes
S_{29}	V_{31}	P_{29}	1	C_{31}	No
S_{31_1}	V_1	P_1	0	$< 1 >$	Yes
S_{31_2}	V_1	P_1	0	$< 1 >$	Yes
S_{31_3}	V_1	P_1	0	$< 1 >$	Yes
S_{96}	V_{31}	P_3	1	C_{31}	No
S_{124_1}	V_1	P_1	0	$< 1 >$	Yes
S_{124_2}	V_1	P_1	0	$< 1 >$	Yes
S_{124_3}	V_1	P_1	0	$< 1 >$	Yes
S_{124_4}	V_1	P_1	0	$< 1 >$	Yes
S_{124_5}	V_1	P_1	0	$< 1 >$	Yes
S_{124_6}	V_1	P_1	0	$< 1 >$	Yes
S_{124_7}	V_1	P_1	0	$< 1 >$	Yes
S_{124_8}	V_1	P_1	0	$< 1 >$	Yes
S_{124_9}	V_1	P_1	0	$< 1 >$	Yes
$S_{124_{10}}$	V_1	P_1	0	$< 1 >$	Yes
S_{155_1}	V_1	P_1	0	$< 1 >$	Yes
S_{155_2}	V_1	P_1	0	$< 1 >$	Yes
S_{155_3}	V_1	P_1	0	$< 1 >$	Yes
S_{186}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

6.1.4 The Linear Group $L_3(7)$

The linear group $L_3(4)$ is of order 1876896 We study simple modules in fields of characteristic $p \in \{2\}$ and present our results below.

Characteristic 2

Table 6.11: The Group $L_3(7)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{32}	P_1	5	QD_{32}	Yes
S_{56}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{152}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{152}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{152}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{288}	V_1	P_1	0	$< 1 >$	Yes
S_{288}	V_1	P_1	0	$< 1 >$	Yes
S_{288}	V_1	P_1	0	$< 1 >$	Yes
S_{288}	V_1	P_1	0	$< 1 >$	Yes
S_{288}	V_1	P_1	0	$< 1 >$	Yes
S_{288}	V_1	P_1	0	$< 1 >$	Yes
S_{342}	V_{16}	P_3	4	Q_{16}	No

Only module in block with defect 4 has source that is not endo-permutation modules. All other modules have sources that are endo-permutation modules.

6.1.5 The Linear Group $L_4(3)$

The linear group $L_3(3)$ is of order $6065280 = 2^7 \cdot 3^6 \cdot 5 \cdot 13$ We study simple modules in fields of characteristic $p \in \{2, 3\}$ and present our results below.

Characteristic 2

Table 6.12: The Group $L_4(3)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{128}	P_1	7	$(D_8 \times D_8) : C_2$	Yes
S_{26_1}	V_{128}	P_6	7	$(D_8 \times D_8) : C_2$	No
S_{26_2}	V_{128}	P_{10}	7	$(D_8 \times D_8) : C_2$	No
S_{38}	V_{128}	P_{10}	7	$(D_8 \times D_8) : C_2$	No
S_{208_1}	V_{128}	P_{15}	7	$(D_8 \times D_8) : C_2$	No
S_{208_2}	V_{128}	P_{19}	7	$(D_8 \times D_8) : C_2$	No
S_{260}	V_{32}	P_1	5	$(C_2 \times C_2 \times C_2) : (C_2 \times C_2)$	Yes
S_{416}	V_4	P_1	2	C_4	Yes
S_{640_1}	V_1	P_1	0	$< 1 >$	Yes
S_{640_2}	V_1	P_1	0	$< 1 >$	Yes
S_{640_3}	V_1	P_1	0	$< 1 >$	Yes
S_{640_4}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 7, have sources that are not endo-permutation modules.

Characteristic 3

Table 6.13: The Group $L_4(3)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation
S_1	V_{729}	P_1	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	Yes
S_6	V_{729}	P_6	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{10}	V_{729}	P_{10}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{10}	V_{729}	P_{10}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{15}	V_{729}	P_{15}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{19}	V_{729}	P_{19}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{44}	V_{729}	P_{44}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{45}	V_{729}	P_{45}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{45}	V_{729}	P_{45}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{69}	V_{729}	P_{69}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{126}	V_{729}	P_{126}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{126}	V_{729}	P_{126}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{156}	V_{729}	P_{156}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{294}	V_{729}	P_{294}	6	$((C_3 \times ((C_3 \times C_3) : C_3)) : C_3) : C_3$	No
S_{729}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 6, have sources that are not endo-permutation modules.

6.1.6 The Linear Group $L_3(3)$

The linear group $L_3(3)$ is of order $5616 = 2^4 \cdot 3^3 \cdot 13$ We study simple modules in fields of characteristic $p \in \{3, 13\}$ and present our results below.

Characteristic 3

Table 6.14: The Group $L_3(3)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{27}	P_1	3	$(C_3 \times C_3) : C_3$	Yes
S_3	V_{27}	P_3	3	$(C_3 \times C_3) : C_3$	No
S_3	V_{27}	P_3	3	$(C_3 \times C_3) : C_3$	No
S_6	V_{27}	P_6	3	$(C_3 \times C_3) : C_3$	No
S_6	V_{27}	P_6	3	$(C_3 \times C_3) : C_3$	No
S_7	V_{27}	P_7	3	$(C_3 \times C_3) : C_3$	No
S_{15}	V_{27}	P_{15}	3	$(C_3 \times C_3) : C_3$	No
S_{15}	V_{27}	P_{15}	3	$(C_3 \times C_3) : C_3$	No
S_{27}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 3, have sources that are not endo-permutation modules.

Characteristic 13

Table 6.15: The Group $L_3(3)$ for characteristic 13

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{13}	P_1	1	C_13	Yes
S_{11}	V_{13}	P_{11}	1	C_{13}	Yes
S_{13}	V_1	P_1	0	$< 1 >$	Yes
S_{16}	V_{13}	P_3	1	C_{13}	No
S_{26_1}	V_1	P_1	0	$< 1 >$	Yes
S_{26_2}	V_1	P_1	0	$< 1 >$	Yes
S_{26_3}	V_1	P_1	0	$< 1 >$	Yes
S_{39}	V_1	P_1	0	$< 1 >$	Yes

Only module S_{16} in the block, with defect 1, has sources that are not endo-permutation modules.

6.1.7 The Linear Group $L_3(4)$

The linear group $L_3(3)$ is of order $20160 = 2^6 \cdot 3^2 \cdot 5 \cdot 7$. It has presentation $< a, b | a^2 = b^4 = (ab)^7 = (ab^2)^5 = (abab^2)^7 = (ababab^2ab^{-1})^5 = 1 >$ We study simple modules in fields of characteristic $p \in \{2, 3, 5, 7\}$ and present our results below.

Characteristic 2

Table 6.16: The Group $L_3(4)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	EP?
S_1	V_{64}	P_1	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	Yes
S_{8_1}	V_{64}	P_8	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{8_2}	V_{64}	P_8	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{9_1}	V_{64}	P_9	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{9_2}	V_{64}	P_9	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{64}	V_1	P_1	0	$< 1 >$	Yes

Here, read EP as Endo-permutation.
All modules that lie in the principal block, with defect 6, have sources that are not endo-permutation modules.

Characteristic 3

Table 6.17: The Group $L_3(4)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_9	P_1	2	$C_3 \times C_3$	Yes
S_{15_1}	V_9	P_6	2	$C_3 \times C_3$	No
S_{15_2}	V_9	P_6	2	$C_3 \times C_3$	No
S_{15_3}	V_9	P_6	2	$C_3 \times C_3$	No
S_{19}	V_9	P_{10}	2	$C_3 \times C_3$	No
S_{45_1}	V_1	P_1	0	$< 1 >$	Yes
S_{45_2}	V_1	P_1	0	$< 1 >$	Yes
S_{63_1}	V_1	P_1	0	$< 1 >$	Yes
S_{63_2}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 2, have sources that are not endo-permutation modules.

Characterisitc 5

Table 6.18: The Group $L_3(4)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{20}	V_1	P_1	0	$< 1 >$	Yes
S_{35_1}	V_1	P_1	0	$< 1 >$	Yes
S_{35_2}	V_1	P_1	0	$< 1 >$	Yes
S_{35_3}	V_1	P_1	0	$< 1 >$	Yes
S_{45_1}	V_1	P_1	0	$< 1 >$	Yes
S_{45_2}	V_1	P_1	0	$< 1 >$	Yes
S_{63}	V_5	P_3	1	C_5	No

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

Characteristic 7

Table 6.19: The Group $L_3(4)$ for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_{19}	V_7	P_5	1	C_7	No
S_{35_1}	V_1	P_1	0	$< 1 >$	Yes
S_{35_2}	V_1	P_1	0	$< 1 >$	Yes
S_{35_3}	V_1	P_1	0	$< 1 >$	Yes
S_{45}	V_7	P_3	1	C_7	No
S_{63_1}	V_1	P_1	0	$< 1 >$	Yes
S_{63_2}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

6.1.8 The Linear Group $L_3(5)$

The linear group $L_3(5)$ is of order $372000 = 2^5 \cdot 3 \cdot 5^3 \cdot 31$. It has presentation $< a, b | a^3 = b^5 = aba^{-1}baba^{-1}b^2ab^{-2}a^{-1}b^2 = abab^{-2}(a^{-1}b^2a^{-1}b^{-2})^3 = 1 >$ We study simple modules in fields of characteristic $p \in \{2, 3, 5, 31\}$ and present our results below.

Characteristic 2

Table 6.20: The Group $L_3(5)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{32}	P_1	5	$(C_4 \times C_4) : C_2$	Yes
S_{30}	V_{16}	P_1	4	$(C_4 \times C_4)$	Yes
S_{96_1}	V_1	P_1	0	$< 1 >$	Yes
S_{96_2}	V_1	P_1	0	$< 1 >$	Yes
S_{96_3}	V_1	P_1	0	$< 1 >$	Yes
S_{96_4}	V_1	P_1	0	$< 1 >$	Yes
S_{96_4}	V_1	P_1	0	$< 1 >$	Yes
S_{96_5}	V_1	P_1	0	$< 1 >$	Yes
S_{96_6}	V_1	P_1	0	$< 1 >$	Yes
S_{96_7}	V_1	P_1	0	$< 1 >$	Yes
S_{96_8}	V_1	P_1	0	$< 1 >$	Yes
S_{96_9}	V_1	P_1	0	$< 1 >$	Yes
S_{124_1}	V_8	P_1	3	C_8	Yes*
S_{124_2}	V_{16}	P_4	4	$(C_4 \times C_4)$	No

Characteristic 3

Table 6.21: The Group $L_3(5)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{30}	V_1	P_1	0	$< 1 >$	Yes
S_{31_1}	V_3	P_1	1	C_3	Yes*
S_{31_2}	V_3	P_1	1	C_3	Yes*
S_{31_3}	V_3	P_1	1	C_3	Yes*
S_{96_1}	V_1	P_1	0	$< 1 >$	Yes
S_{96_2}	V_1	P_1	0	$< 1 >$	Yes
S_{96_3}	V_1	P_1	0	$< 1 >$	Yes
S_{96_4}	V_1	P_1	0	$< 1 >$	Yes
S_{96_5}	V_1	P_1	0	$< 1 >$	Yes
S_{96_6}	V_1	P_1	0	$< 1 >$	Yes
S_{96_7}	V_1	P_1	0	$< 1 >$	Yes
S_{96_8}	V_1	P_1	0	$< 1 >$	Yes
S_{96_9}	V_1	P_1	0	$< 1 >$	Yes
$S_{96_{10}}$	V_1	P_1	0	$< 1 >$	Yes
S_{124_1}	V_3	P_1	1	C_3	Yes*
S_{124_2}	V_3	P_1	1	C_3	Yes*
S_{124_3}	V_3	P_2	1	C_3	No
S_{124_4}	V_3	P_2	1	C_3	No
S_{124_5}	V_3	P_1	1	C_3	Yes*
S_{124_6}	V_3	P_1	1	C_3	Yes*
S_{186}	V_1	P_1	0	$< 1 >$	Yes

Characteristic 5

Table 6.22: The Group $L_3(5)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{125}	P_1	3	$(C_5 \times C_5) : C_5$	Yes
S_{3_1}	V_{125}	P_3	3	$(C_5 \times C_5) : C_5$	No
S_{3_2}	V_{125}	P_3	3	$(C_5 \times C_5) : C_5$	No
S_{6_1}	V_{125}	P_6	3	$(C_5 \times C_5) : C_5$	No
S_{6_2}	V_{125}	P_6	3	$(C_5 \times C_5) : C_5$	No
S_8	V_{125}	P_8	3	$(C_5 \times C_5) : C_5$	No
S_{10_1}	V_{125}	P_{10}	3	$(C_5 \times C_5) : C_5$	No
S_{10_2}	V_{125}	P_{10}	3	$(C_5 \times C_5) : C_5$	No
S_{15_1}	V_{125}	P_{15}	3	$(C_5 \times C_5) : C_5$	No
S_{15_2}	V_{125}	P_{15}	3	$(C_5 \times C_5) : C_5$	No
S_{15_3}	V_{125}	P_{15}	3	$(C_5 \times C_5) : C_5$	No
S_{15_4}	V_{125}	P_{15}	3	$(C_5 \times C_5) : C_5$	No
S_{18_1}	V_{125}	P_{18}	3	$(C_5 \times C_5) : C_5$	No
S_{18_2}	V_{125}	P_{18}	3	$(C_5 \times C_5) : C_5$	No
S_{19}	V_{125}	P_{19}	3	$(C_5 \times C_5) : C_5$	No
S_{35_1}	V_{125}	P_{35}	3	$(C_5 \times C_5) : C_5$	No
S_{35_2}	V_{125}	P_{35}	3	$(C_5 \times C_5) : C_5$	No
S_{39_1}	V_{125}	P_{39}	3	$(C_5 \times C_5) : C_5$	No
S_{39_2}	V_{125}	P_{39}	3	$(C_5 \times C_5) : C_5$	No
S_{60_1}	V_{125}	P_{60}	3	$(C_5 \times C_5) : C_5$	No
S_{60_2}	V_{125}	P_{60}	3	$(C_5 \times C_5) : C_5$	No
S_{63}	V_{125}	P_{63}	3	$(C_5 \times C_5) : C_5$	No
S_{90_1}	V_{125}	P_{90}	3	$(C_5 \times C_5) : C_5$	No
S_{90_1}	V_{125}	P_{90}	3	$(C_5 \times C_5) : C_5$	No
S_{125}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 3, have sources that are not endo-permutation modules.

Characteristic 31

Table 6.23: The Group $L_3(5)$ for characteristic 31

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{31}	P_1	1	C_{31}	Yes
S_{29}	V_{31}	P_{29}	1	C_{31}	No
S_{31_1}	V_1	P_1	0	$< 1 >$	Yes
S_{31_2}	V_1	P_1	0	$< 1 >$	Yes
S_{31_3}	V_1	P_1	0	$< 1 >$	Yes
S_{96}	V_{31}	P_3	1	C_{31}	No
S_{124_1}	V_1	P_1	0	$< 1 >$	Yes
S_{124_2}	V_1	P_1	0	$< 1 >$	Yes
S_{124_3}	V_1	P_1	0	$< 1 >$	Yes
S_{124_4}	V_1	P_1	0	$< 1 >$	Yes
S_{124_5}	V_1	P_1	0	$< 1 >$	Yes
S_{124_6}	V_1	P_1	0	$< 1 >$	Yes
S_{124_7}	V_1	P_1	0	$< 1 >$	Yes
S_{124_8}	V_1	P_1	0	$< 1 >$	Yes
S_{124_9}	V_1	P_1	0	$< 1 >$	Yes
$S_{124_{10}}$	V_1	P_1	0	$< 1 >$	Yes
S_{155_1}	V_1	P_1	0	$< 1 >$	Yes
S_{155_2}	V_1	P_1	0	$< 1 >$	Yes
S_{155_3}	V_1	P_1	0	$< 1 >$	Yes
S_{186}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

6.1.9 The Linear Group $L_3(7)$

The linear group $L_3(7)$ is of order 1876896. We study simple modules in fields of characteristic $p \in \{2\}$ and present our results below.

Characteristic 2

Table 6.24: The Group $L_3(7)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{32}	P_1	5	QD_{32}	Yes
S_{56}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{152_1}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{152_2}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{152_3}	V_4	P_1	2	$C_2 \times C_2$	Yes*
S_{288_1}	V_1	P_1	0	$< 1 >$	Yes
S_{288_2}	V_1	P_1	0	$< 1 >$	Yes
S_{288_3}	V_1	P_1	0	$< 1 >$	Yes
S_{288_4}	V_1	P_1	0	$< 1 >$	Yes
S_{288_5}	V_1	P_1	0	$< 1 >$	Yes
S_{288_6}	V_1	P_1	0	$< 1 >$	Yes
S_{342}	V_{16}	P_3	4	Q_{16}	No

All modules that lie in the block, with defect 4, have sources that are not endo-permutation modules.

6.2 Unitary Groups

6.2.1 The Unitary Group $U_3(3)$

The unitary group $U_3(3)$ is of order $6048 = 2^5 \cdot 3^3 \cdot 7$. It has presentation $< a, b | a^2 = b^6 = (ab)^7 = [a, (ab^2)^3] = b^3[b^2, ab^3a]^2 = 1 >$ We study simple modules in fields of characteristic $p \in \{2, 3, 5, 7\}$ and present our results below.

Characteristic 2

Table 6.25: The Group $U_3(3)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{32}	P_1	5	$(C_4 \times C_4) : C_2$	Yes
S_6	V_{32}	P_6	5	$(C_4 \times C_4) : C_2$	No
S_{14}	V_{32}	P_{14}	5	$(C_4 \times C_4) : C_2$	No
S_{32_1}	V_1	P_1	0	$< 1 >$	Yes
S_{32_2}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 5, have sources that are not endo-permutation modules.

Characteristics 3

Table 6.26: The Group $U_3(3)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{27}	P_1	3	$(C_3 \times C_3) : C_3$	Yes
S_{3_1}	V_{27}	P_3	3	$(C_3 \times C_3) : C_3$	No
S_{3_2}	V_{27}	P_3	3	$(C_3 \times C_3) : C_3$	No
S_{6_1}	V_{27}	P_6	3	$(C_3 \times C_3) : C_3$	No
S_{6_2}	V_{27}	P_6	3	$(C_3 \times C_3) : C_3$	No
S_7	V_{27}	P_7	3	$(C_3 \times C_3) : C_3$	No
S_{15_1}	V_{27}	P_{15}	3	$(C_3 \times C_3) : C_3$	No
S_{15_2}	V_{27}	P_{15}	3	$(C_3 \times C_3) : C_3$	No
S_{27}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 3, have sources that are not endo-permutation modules.

Characteristic 7

Table 6.27: The Group $U_3(3)$ for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_6	V_7	P_6	1	C_7	No
S_{7_1}	V_1	P_1	0	$< 1 >$	Yes
S_{7_2}	V_1	P_1	0	$< 1 >$	Yes
S_{7_3}	V_1	P_1	0	$< 1 >$	Yes
S_{14}	V_1	P_1	0	$< 1 >$	Yes
S_{21_1}	V_1	P_1	0	$< 1 >$	Yes
S_{21_2}	V_1	P_1	0	$< 1 >$	Yes
S_{21_3}	V_1	P_1	0	$< 1 >$	Yes
S_{26}	V_7	P_5	1	C_7	No
S_{28_1}	V_1	P_1	0	$< 1 >$	Yes
S_{28_2}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

6.2.2 The Unitary Group $U_3(4)$

The unitary group $U_3(4)$ is of order $62400 = 2^6 \cdot 3 \cdot 5^2 \cdot 13$. It has presentation $< a, b | a^2 = b^3 = (ab)^{13} = [a, b]^5 = [a, babab]^3 = 1 >$ We study simple modules in fields of characteristic $p \in \{2, 3, 5, 13\}$ and present our results below.

Characteristic 2

Table 6.28: The Group $U_3(4)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	EP?
S_1	V_{64}	P_1	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	Yes
S_{3_1}	V_{64}	P_3	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{3_2}	V_{64}	P_3	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{3_3}	V_{64}	P_3	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{3_4}	V_{64}	P_3	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{8_1}	V_{64}	P_8	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{8_2}	V_{64}	P_8	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{9_1}	V_{64}	P_9	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{9_2}	V_{64}	P_9	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{9_3}	V_{64}	P_9	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{9_4}	V_{64}	P_9	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{24_1}	V_{64}	P_{24}	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{24_2}	V_{64}	P_{24}	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{24_3}	V_{64}	P_{24}	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{24_4}	V_{64}	P_{24}	6	$(C_2 \times C_2).(C_2 \times C_2 \times C_2 \times C_2)$	No
S_{64}	V_1	P_1	0	$< 1 >$	Yes

Here, read EP as Endo-permutation All modules that lie in the principal block, with defect 6, have sources that are not endo-permutation modules.

Characteristic 3

Table 6.29: The Group $U_3(4)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_3	P_1	1	C_3	Yes
S_{12}	V_1	P_1	0	$< 1 >$	Yes
S_{13_1}	V_3	P_1	0	C_3	Yes*
S_{13_2}	V_3	P_1	0	C_3	Yes*
S_{13_3}	V_3	P_1	0	C_3	Yes*
S_{13_4}	V_3	P_1	1	C_3	Yes*
S_{39_1}	V_1	P_1	0	$< 1 >$	Yes
S_{39_2}	V_1	P_1	0	$< 1 >$	Yes
S_{52_1}	V_3	P_1	0	C_3	Yes*
S_{52_2}	V_3	P_1	1	C_3	Yes*
S_{52_3}	V_3	P_1	0	C_3	Yes*
S_{52_4}	V_3	P_1	0	C_3	Yes*
S_{64}	V_3	P_1	0	C_3	Yes*
S_{75_1}	V_1	P_1	0	$< 1 >$	Yes
S_{75_2}	V_1	P_1	0	$< 1 >$	Yes
S_{75_3}	V_1	P_1	0	$< 1 >$	Yes
S_{75_4}	V_1	P_1	0	$< 1 >$	Yes

All modules have sources that are endo-permutation modules.

Characteristic 5

Table 6.30: The Group $U_3(4)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{25}	P_1	2	$C_5 \times C_5$	Yes
S_{12}	V_{25}	P_{12}	2	$C_5 \times C_5$	No
S_{39}	V_{25}	P_{14}	2	$C_5 \times C_5$	No
S_{65}	V_5	P_1	2	$C_5 \times C_5$	Yes*
S_{75_1}	V_1	P_1	0	$< 1 >$	Yes
S_{75_2}	V_1	P_1	0	$< 1 >$	Yes
S_{75_3}	V_1	P_1	0	$< 1 >$	Yes
S_{75_4}	V_1	P_1	0	$< 1 >$	Yes

All modules; except S_{65}; that lie in the principal block, with defect 2, have sources that are not endo-permutation modules.

Characteristic 13

Table 6.31: The Group $U_3(4)$ for characteristic 13

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{13}	P_1	1	C_{13}	Yes
S_{12}	V_{13}	P_{12}	0	C_{13}	No
S_{13_1}	V_1	P_{13}	0	$< 1 >$	Yes
S_{13_2}	V_1	P_{13}	0	$< 1 >$	Yes
S_{13_3}	V_1	P_{13}	0	$< 1 >$	Yes
S_{13_4}	V_1	P_{13}	0	$< 1 >$	Yes
S_{39_1}	V_1	P_{39}	0	$< 1 >$	Yes
S_{39_2}	V_1	P_{39}	0	$< 1 >$	Yes
S_{52_1}	V_1	P_{52}	0	$< 1 >$	Yes
S_{52_2}	V_1	P_{52}	0	$< 1 >$	Yes
S_{52_3}	V_1	P_{52}	0	$< 1 >$	Yes
S_{52_4}	V_1	P_{52}	0	$< 1 >$	Yes
S_{63}	V_{13}	P_{11}	1	C_{13}	No
S_{65_1}	V_1	P_{65}	0	$< 1 >$	Yes
S_{65_2}	V_1	P_{65}	0	$< 1 >$	Yes
S_{65_3}	V_1	P_{65}	0	$< 1 >$	Yes
S_{65_4}	V_1	P_{65}	0	$< 1 >$	Yes
S_{65_5}	V_1	P_{65}	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

6.2.3 The Unitary Group $U_4(2)$

The unitary group $U_4(2)$ is of order $25920 = 2^6 \cdot 3^4 \cdot 5$. It has presentation $< a, b | a^2 = b^5 = (ab)^9 = [a, b]^3 = [a, bab]^2 = 1 >$ We study simple modules in fields of characteristic $p \in \{2, 3, 5\}$ and present our results below.

Characteristic 2

Table 6.32: The Group $U_4(2)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{64}	P_1	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	Yes
S_4	V_{64}	P_4	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_4	V_{64}	P_4	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_6	V_{64}	P_6	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{14}	V_{64}	P_{14}	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{20_1}	V_{64}	P_{20}	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{20_2}	V_{64}	P_{20}	6	$((C_2 \times C_2 \times C_2 \times C_2) : C_2) : C_2$	No
S_{64}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 6, have sources that are not endo-permutation modules.

Characteristic 3

Table 6.33: The Group $U_4(2)$ for characteristic 3

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{81}	P_1	4	$(C_3 \times C_3 \times C_3) : C_3$	Yes
S_5	V_{81}	P_5	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{10}	V_{81}	P_{10}	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{14}	V_{81}	P_{14}	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{25}	V_{81}	P_{25}	4	$(C_3 \times C_3 \times C_3) : C_3$	No
S_{81}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 4, have sources that are not endo-permutation modules.

Characteristic 5

Table 6.34: The Group $U_4(2)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_5	V_1	P_1	0	$< 1 >$	Yes
S_5	V_1	P_1	0	$< 1 >$	Yes
S_6	V_5	P_1	1	C_5	Yes*
S_{10}	V_1	P_1	0	$< 1 >$	Yes
S_{10}	V_1	P_1	0	$< 1 >$	Yes
S_{15}	V_1	P_1	0	$< 1 >$	Yes
S_{15}	V_1	P_1	0	$< 1 >$	Yes
S_{20}	V_1	P_1	0	$< 1 >$	Yes
S_{23}	V_5	P_3	1	C_5	No
S_{30}	V_1	P_1	0	$< 1 >$	Yes
S_{30}	V_1	P_1	0	$< 1 >$	Yes
S_{30}	V_1	P_1	0	$< 1 >$	Yes
S_{40}	V_1	P_1	0	$< 1 >$	Yes
S_{40}	V_1	P_1	0	$< 1 >$	Yes
S_{45}	V_1	P_1	0	$< 1 >$	Yes
S_{45}	V_1	P_1	0	$< 1 >$	Yes
S_{58}	V_5	P_3	1	C_5	No
S_{60}	V_1	P_1	0	$< 1 >$	Yes

All modules;except S_6; that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

6.3 Suzuki Group

6.3.1 The Suzuki Group $Sz(8)$

The suzuki group $Sz(8)$ is of order $29120 = 2^6 \cdot 5 \cdot 7 \cdot 13$. It has presentation $< a, b | a^2 = b^5 = (ab)^9 = [a, b]^3 = [a, bab]^2 = 1 >$ We study simple modules in fields of characteristic $p \in \{2, 5, 7, 13\}$ and present our results below.

Characteristic 2

Table 6.35: The Group $Sz(8)$ for characteristic 2

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{64}	P_1	6	$(C_2 \times C_2 \times C_2).(C_2 \times C_2 \times C_2)$	Yes
S_{4_1}	V_{64}	P_4	6	$(C_2 \times C_2 \times C_2).(C_2 \times C_2 \times C_2)$	No
S_{4_2}	V_{64}	P_4	6	$(C_2 \times C_2 \times C_2).(C_2 \times C_2 \times C_2)$	No
S_{4_3}	V_{64}	P_6	6	$(C_2 \times C_2 \times C_2).(C_2 \times C_2 \times C_2)$	No
S_{16_1}	V_{64}	P_{16}	6	$(C_2 \times C_2 \times C_2).(C_2 \times C_2 \times C_2)$	No
S_{16_2}	V_{64}	P_{16}	6	$(C_2 \times C_2 \times C_2).(C_2 \times C_2 \times C_2)$	No
S_{16_3}	V_{64}	P_{16}	6	$(C_2 \times C_2 \times C_2).(C_2 \times C_2 \times C_2)$	No
S_{64}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

Characteristic 5

Table 6.36: The Group $Sz(8)$ for characteristic 5

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_5	P_1	1	C_5	Yes
S_{14_1}	V_5	P_4	1	C_5	No
S_{14_2}	V_5	P_4	1	C_5	No
S_{35_1}	V_1	P_1	0	$< 1 >$	Yes
S_{35_2}	V_1	P_1	0	$< 1 >$	Yes
S_{35_3}	V_1	P_1	0	$< 1 >$	Yes
S_{63}	V_5	P_3	0	C_5	No
S_{65_1}	V_1	P_1	0	$< 1 >$	Yes
S_{65_2}	V_1	P_1	0	$< 1 >$	Yes
S_{65_3}	V_1	P_1	0	$< 1 >$	Yes

All modules that lie in the principal block, with defect 1, have sources that are not endo-permutation modules.

Characteristic 7

Table 6.37: The Group $Sz(8)$ for characteristic 7

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_7	P_1	1	C_7	Yes
S_{14_1}	V_1	P_1	0	$< 1 >$	Yes
S_{14_2}	V_1	P_1	0	$< 1 >$	Yes
S_{35_1}	V_1	P_1	0	$< 1 >$	Yes
S_{35_2}	V_1	P_1	0	$< 1 >$	Yes
S_{35_3}	V_1	P_1	0	$< 1 >$	Yes
S_{64}	V_7	P_1	0	C_7	Yes*
S_{91}	V_1	P_1	0	$< 1 >$	Yes

All modules have sources that are endo-permutation modules.

Characteristic 13

Table 6.38: The Group $Sz(8)$ for characteristic 13

Module	Vertex	Source	Defect	Defect Group	Endo-permutation?
S_1	V_{13}	P_1	1	C_{13}	Yes
S_{14_1}	V_{13}	P_1	1	C_{13}	Yes*
S_{14_2}	V_{13}	P_1	1	C_{13}	Yes*
S_{35}	V_{13}	P_9	1	C_{13}	No
S_{65_1}	V_1	P_1	0	$< 1 >$	Yes
S_{65_2}	V_1	P_1	0	$< 1 >$	Yes
S_{65_3}	V_1	P_1	0	$< 1 >$	Yes
S_{91}	V_1	P_1	0	$< 1 >$	Yes

Chapter 7

Conclusion

In this chapter we summarize our observations from the data collected.

7.1 Observations

1. Most of the cases where the source is endo-permutation, it is the trivial module.

   (a) Of these cases, in most cases, the modules belong to a block of defect 0.

   (b) In other cases, source module was explicitly calculated.

2. The higher the defect of a block, higher the chances of it implying that modules liyng in the block have sources that are not endo-permutation modules.

3. The higher the characterisitc of the field, higher the chances of it all that modules have sources that are not endo-permutation modules even with a small defect.

4. In cases where our computer program returns a false value for IsPermutationModule on the restriction, source module is endo-permutation if and only if it is the trivial 1-dimensional module.

5. All modules lying in blocks of defect $d = 7$ have sources that are not endo-permutation modules.

6. Almost all modules lying in blocks of defect $d = 6$ have sources that are not endo-permutation modules.

7. Almost all modules considered in a field of characteristic greater than 7 have sources that are not endo-permutation modules.

Chapter 8

Appendix: Functions Used

We give a list of commands in MAGMA and GAP that were useful for us. [Cra] [BCP97]

1. `StructureDescription(G)`

   The method for StructureDescription exhibits a structure of the given group G to some extent, using the strategy outlined below. The idea is to return a possibly short string which gives some insight in the structure of the considered group. This is a GAP command.

2. ```
   IrreducibleModules(G, K : parameters) : Grp, Fld -> SeqEnum
   AbsolutelyIrreducibleModules(G, K : parameters) :
   Grp, Fld -> SeqEnum
   ```

   were used these to get all the irreducible KG-modules.

3. ```
   IsPermutationModule(M) : ModRng -> BoolElt
   ```

   Returns true if and only if the generators of the matrix algebra A are permutation matrices, for a given A-module M.

4. ```
   Vertex(M : parameters) : ModGrp -> Grp
   ```

   Returns a representative V of the conjugacy class of vertices of M, V a subgroup of G where M is a K[G]-module. It is necessary that M be an simple K[G]-module, where G is a finite group, and K is a finite field. These conditions are not checked by the function.

5. ```
   Source(M : parameters) : ModGrp -> ModGrp, ModGrp
   ```

> Returns a source module S for the simple G-module M. The group of S is a vertex for M. The second return value is the Green correspondent of M. The parameter H is as for the Vertex command above, and is passed to it.

In all the cases, it is important to choose a field big enough such that the modules can decompose completely in the chosen field. It is possible that a field big enough for a group might not be big enough for its subgroups and hence, we may calculate wrong source. We have taken this into account while calculating sources of all the modules we encountered and made sure that, in each case, our field was big enough.

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